# 2023 AUSTRALIAN SCIENCE OLYMPIAD EXAM

## PHYSICS

**TO BE COMPLETED BY THE STUDENT. USE CAPITAL LETTERS.**

**First Name: ……………….….….………… Last Name………………………………….**

**Date of Birth: ……/…..../...….**

**□ Male □ Female □ Unspecified Year 10 □ Year 11 □ Other: …….**

**Name of School: ……………………………………….………………..State: ……...**

**Examiners Use Only:**

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2023 AUSTRALIAN SCIENCE OLYMPIAD EXAM

# PHYSICS

***Time Allowed***

***Reading Time: 10 minutes***
***Examination Time: 120 minutes***

## INSTRUCTIONS

- *Attempt all questions in ALL sections of this paper.*
- Permitted materials: non-programmable, non-graphical calculator, pens, pencils, erasers and a ruler.
- Marks will not be deducted for incorrect answers.
- Ensure that diagrams are clear and labelled
- All numerical answers must have correct units

## MARKS

| | |
|---|---|
| ▪ Section A: Amusing Airport Adventures | 13 marks |
| ▪ Section B: Rainy Day Radar | 38 marks |
| ▪ Section C: Bounce-back Beam Biopsy | 37 marks |
| ▪ Section D: Seeing Secchis | 21 marks |
| ▪ Section E: Trailer-Towing Truck | 9 marks |
| **TOTAL** | **118 marks** |

***Integrity of Competition***

*If there is evidence of collusion or other academic dishonesty, students will be disqualified. Markers' decisions are final.*

## Section A: Amusing Airport Adventures

### Suggested Time: 15 minutes

Some students are wandering around an airport waiting for their flight.
**This question deals with kinematics only, so there is no need to consider resistive forces such as drag.**

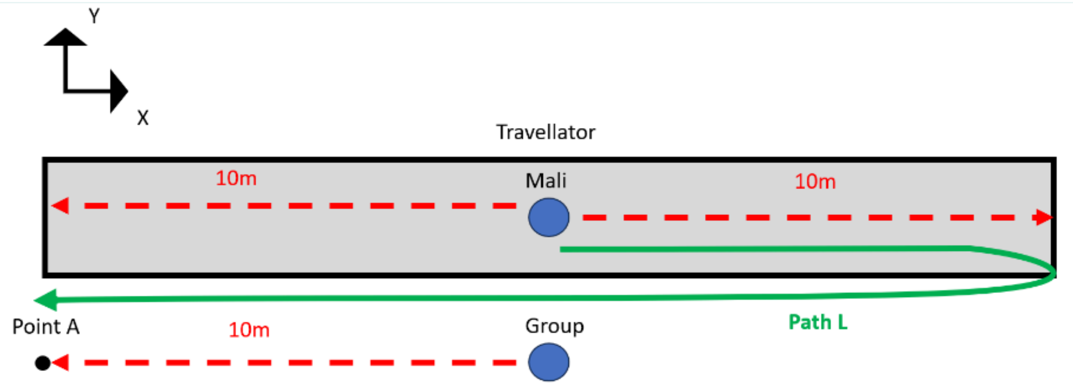
A group of students are walking at a speed of 1.0m/s. They walk alongside a travellator which is moving at a speed of 0.70m/s.

1. One of the students, Mali, decides to walk on the travellator which is moving in the same direction as the group of students. How fast must Mali walk to move at the same speed as the rest of the group? (1 mark)

   Note when we refer to Mali's walking speed we mean with respect to the ground/travellator beneath them.

2. The group stops to read a sign. Mali is still on the travellator. How fast must Mali walk now to stay with the group? (1 mark)

3. The group moves at 1.0m/s towards the left. The travellator moves at 0.70m/s towards the left. Mali walks at a constant speed v relative to the ground/travellator beneath them. Mali wants to take path L and arrive at point A at the same time as the rest of the group. At what speed should Mali move? Ignore any distance travelled in the y-direction for this problem. (4 marks)

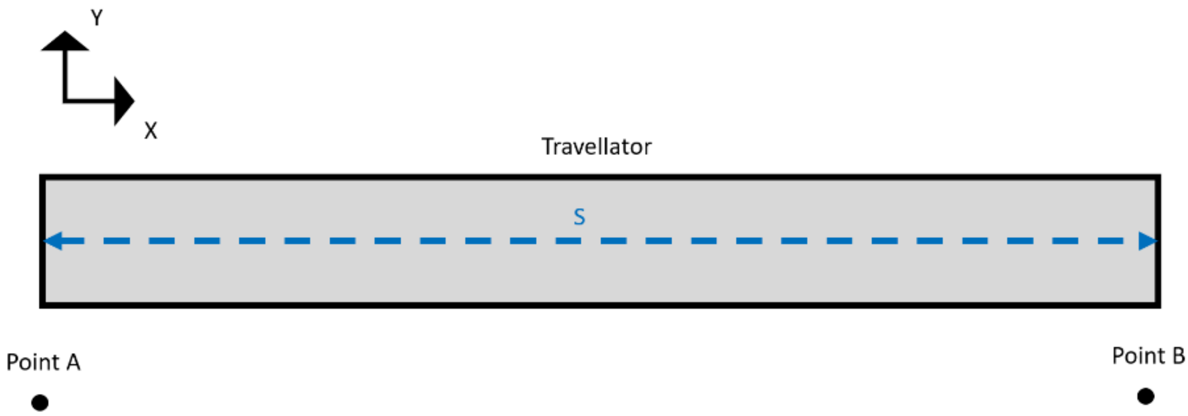


4. You may ignore the y-direction for this question; it exists simply for the clarity of the diagram. Imagine Mali and the group start in line with point A. They walk to the end of the travellator (point B) and then return. Mali completes the entire journey on the travellator. The group does the entire journey on the ground.

   Assume the group walking speed is u. You may not assume that u = 1.0m/s. Assume that the travellator speed is w. You may not assume that w=0.70m/s. Assume the travellator has length S.

   Mali walks at a constant speed, v (relative to the ground beneath them), for the entire journey. Determine speed v such that Mali and the group return at the same time. Give your answer as an algebraic expression.

   Explore the limiting behaviours of your equation. Explain what happens to v when u is very large compared to w, and what happens when w is very large compared to u. What happens to v when S is increased? Does the behaviour of your equation make sense in these scenarios? Why? (7 marks)



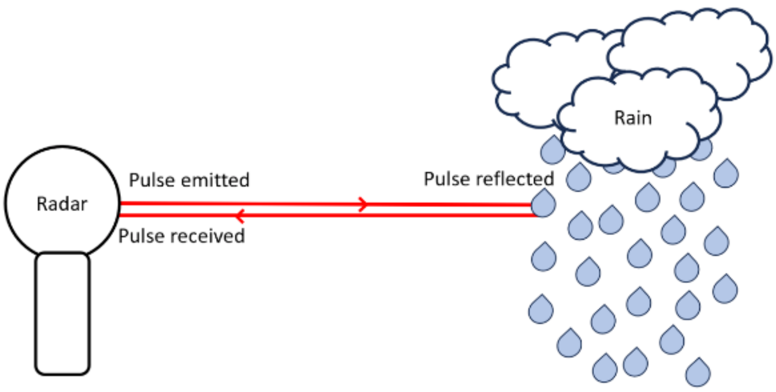
**Section B: Rainy Day Radar**

**Suggested time: 30 minutes**

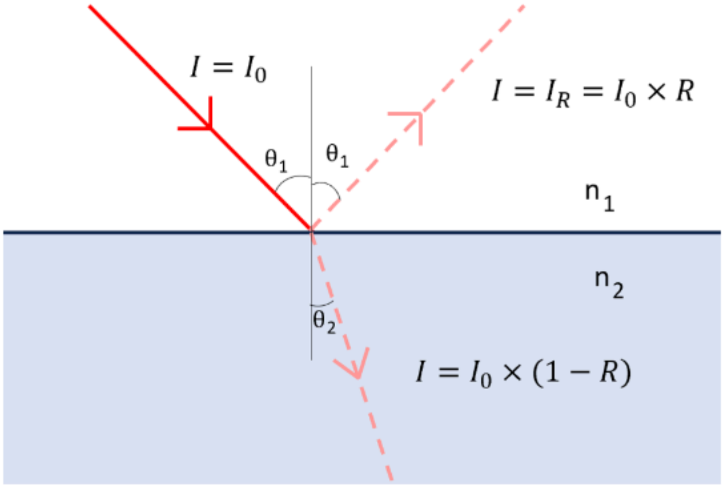
A weather radar (pictured below) is a device used to identify the location and intensity of rain in surrounding areas by sending out a pulse of electromagnetic radiation. This question looks at a simplified model of a weather radar which sends out pulses horizontally to reduce errors from detecting clouds and higher atmosphere precipitation which never reaches the ground. It also neglects the effects of scattering and absorption by precipitation.

A BOM weather radar in Newdegate

1. To determine the location of the rain, the radar uses time of flight. Because the pulse travels at the speed of light ($3.00 \times 10^8$m/s), the time between when the pulse is emitted and received back by the radar can be used to determine the distance from the radar to the rain.




The time between when a pulse is emitted and returned is 0.32 ms. How far away is the rain from the radar? (1 mark)




When the pulse encounters an interface, some radiation is reflected and some is transmitted. Every material has a refractive index (n), which is the ratio of the speed of light in a vacuum to the speed in the material. The proportion of light which is reflected depends both on the refractive indices of the materials on either side of the interface and the angle the light is incident. Assuming the light is incident normal to the surface, the proportion of light which is reflected can be calculated with the equation:

$$R = \frac{I_R}{I_0} = \left(\frac{n_2 - n_1}{n_2 + n_1}\right)^2.$$

2. Using the data below, calculate the percentage of normally incident light which is reflected for a water droplet. (1 mark)

   Refractive index of air = 1.00
   Refractive index of water = 1.33
   Refractive index of ice = 1.31

3. Using the data below, calculate the percentage of normally incident light which is reflected for a solid hailstone. (1 mark)

   Refractive index of air = 1.00
   Refractive index of water = 1.33
   Refractive index of ice = 1.31

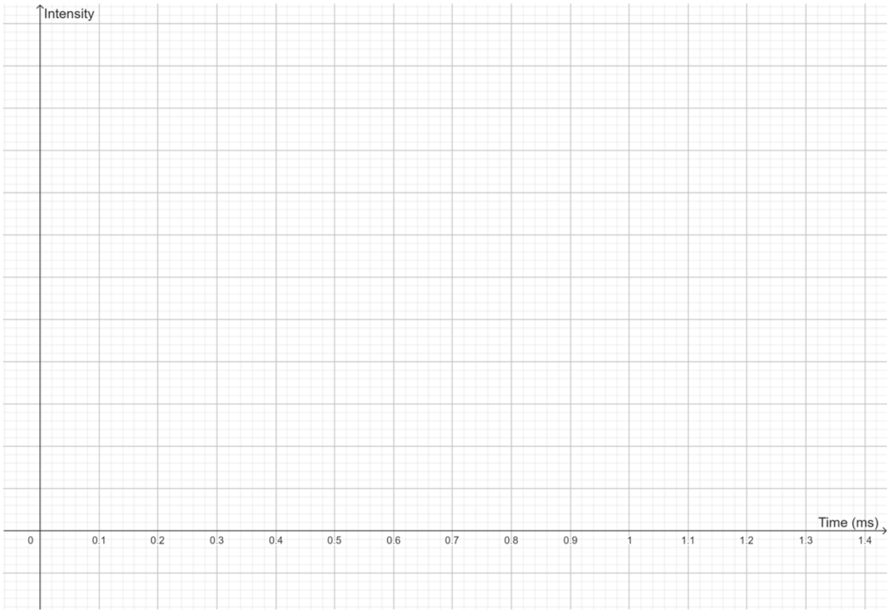
4. Often hailstones form rapidly. This means pockets of air get trapped between thin layers of ice, and the hailstone appears cloudy.

   Describe how this process changes the percentage of light that is reflected by a hailstone compared to what was previously calculated. (1 mark)

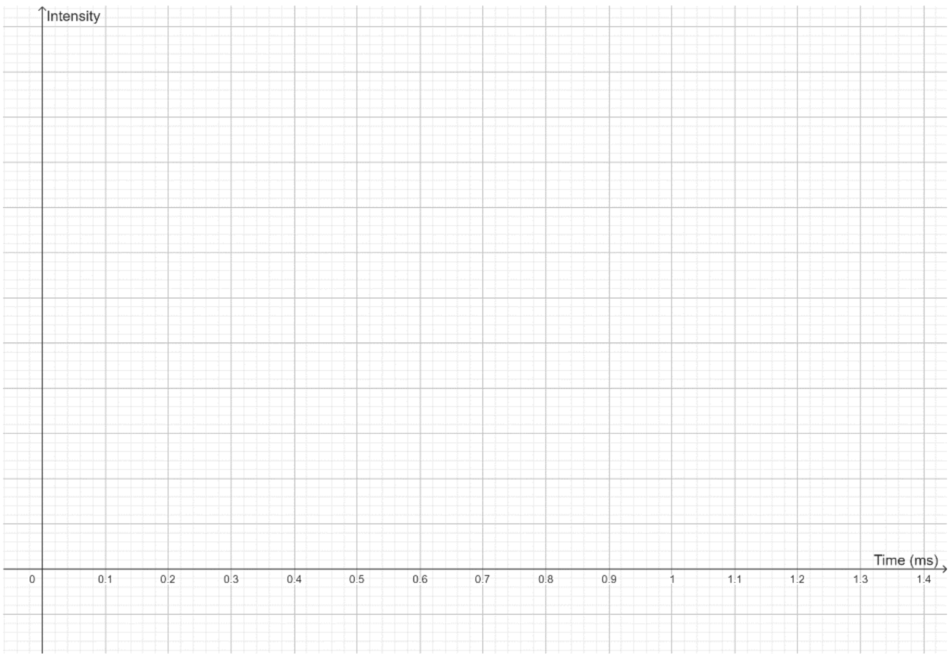
5. If the pulse travels through 50 air pockets, what proportion of its intensity will be reflected? Ignore the wave nature of light for this question. (4 marks)

6. How can the radar use the intensity of the reflected pulse to determine the type and intensity of precipitation? (2 marks)

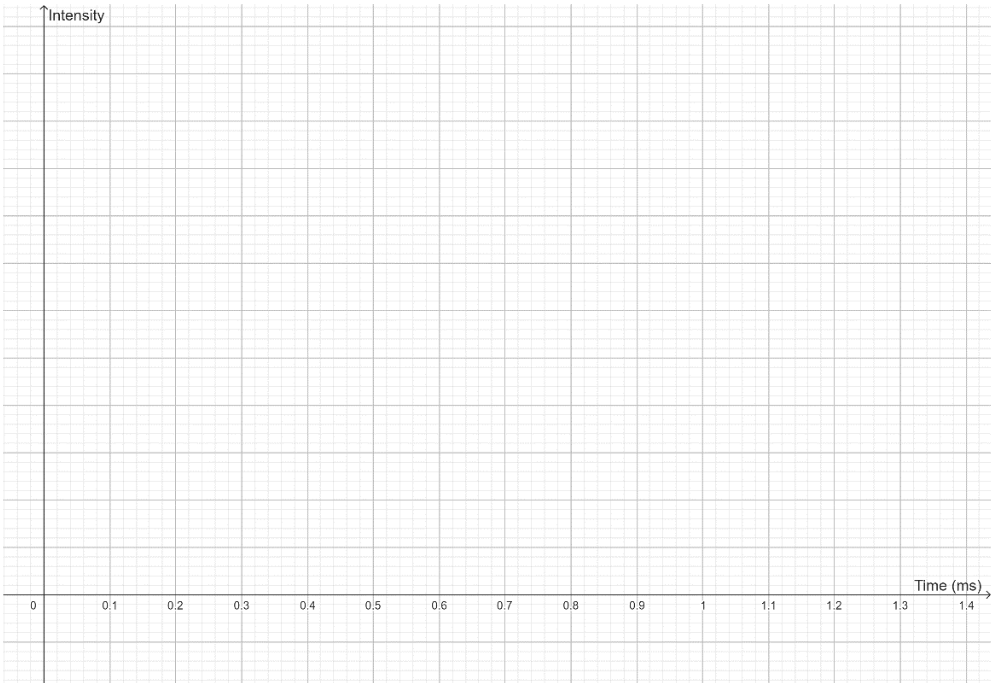
7. Sketch and **justify** the intensity vs time graphs for the pulse received by the radar for two identical small patches of rain which are located 15 km and 150 km from the radar. Take time=0 as the time when the pulse is emitted from the radar, and plot the intensities received by the radar for each case separately but on the same graph rather than a superposition of the pulses. (3 marks)



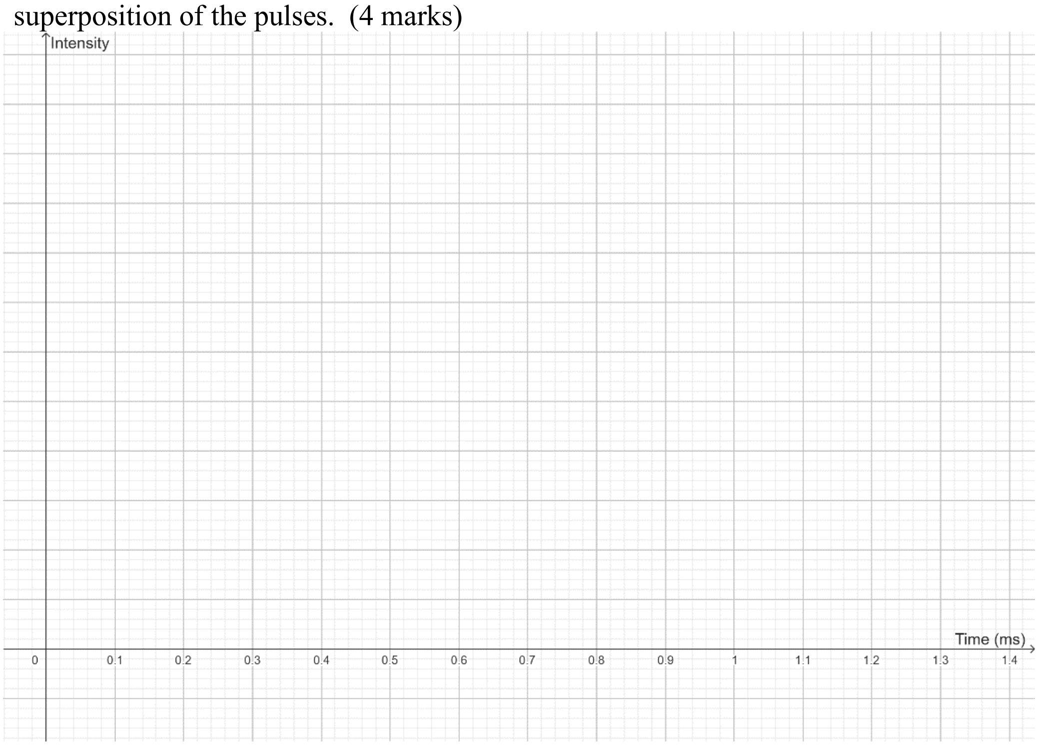

8. Sketch and **justify** the intensity vs time graphs for the pulse received by the radar for two identical small patches of hail and rain each located 50km from the radar. Take time=0 as the time when the pulse is emitted from the radar, and plot the intensities received by the radar for each case separately but on the same graph rather than a superposition of the pulses. (3 marks)



9. Sketch and **justify** the intensity vs time graphs for the pulse received by the radar for a small heavy patch of rain and a small light patch of rain each located 50 km from the radar. Take time=0 as the time when the pulse is emitted from the radar, and plot the intensities received by the radar for each case separately but on the same graph rather than a superposition of the pulses. (3 marks)

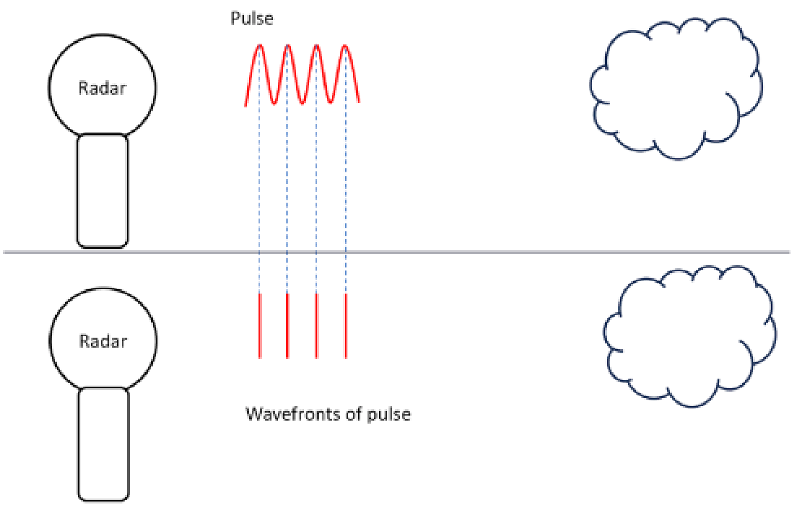



10. Sketch and **justify** the intensity vs time graphs for the pulse received by the radar for a small, dense patch of rain and a larger, less dense band of rain with their centres each located 50 km from the radar. Take time=0 as the time when the pulse is emitted from the radar, and plot the intensities received by the radar for each case separately but on the same graph rather than a superposition of the pulses. (4 marks)

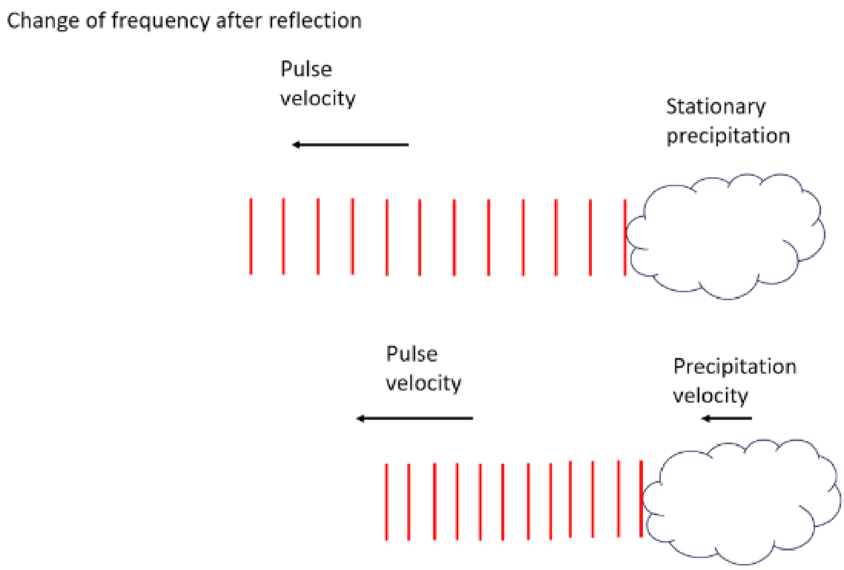


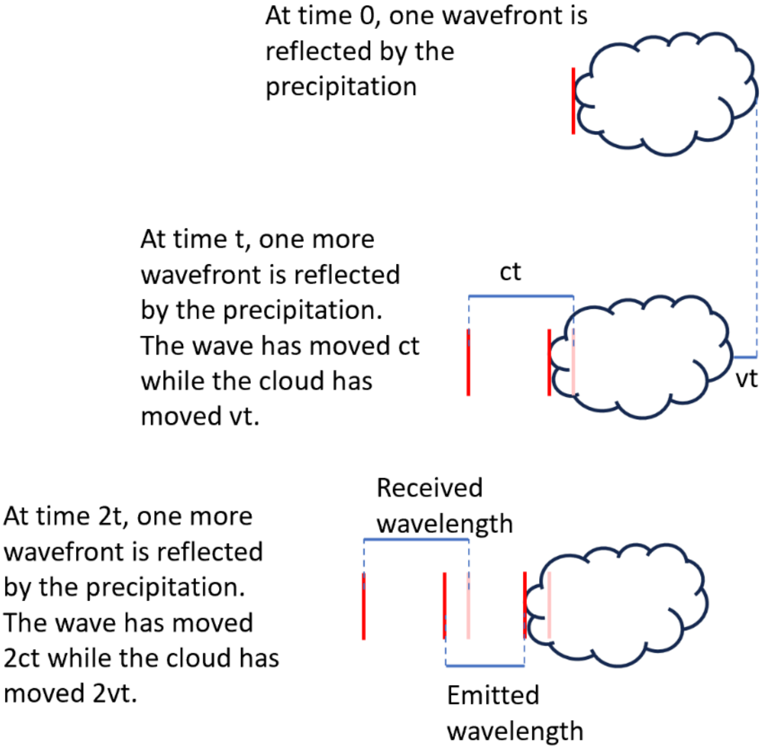
11. The weather radar sometimes displays the signal of patches of light rain behind bands of heavy rain, even when this light rain is not present. Explain what causes this, including any relevant diagrams. (5 marks)

Some very sensitive weather radars are also able to detect changes in frequency between when the pulse is emitted and received. The frequency (Hz) is the number of waves that pass through a fixed point in one second. This means that the radar can be used to determine the speed of the wind and precipitation.




When the pulse is incident on moving precipitation, the precipitation moves slightly between receiving each wavefront and then moves again slightly between reflecting each wavefront. This means that the wavelength (distance between wavefronts, $\lambda$) of radiation which it reflects is different to the wavelength of radiation emitted by the radar. Since the speed of light $c = 2.997924580 \times 10^8$ m/s is constant in air and $c = f\lambda$, a shorter wavelength leads to a higher frequency ($f$) and a longer wavelength to a lower frequency.






In general, the observed frequency due to relative motion between the radar and cloud at speeds much lower than the speed of light can be calculated with the formula:

$$f' = f\frac{v \pm v_O}{v \pm v_S},$$

where:

- $f$ is the original frequency,
- $f'$ is the new frequency,
- $v$ is the speed of light,
- $v_O$ is the speed of the observer, and
- $v_S$ is the speed of the source.

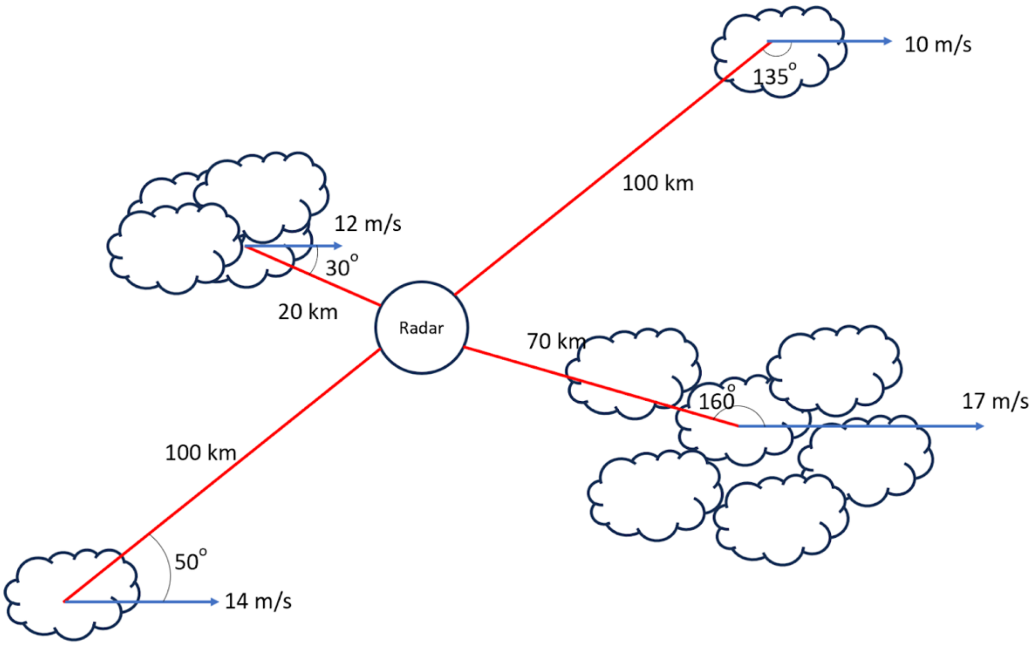
12. Explain why the frequency received by the radar is:

$$f' = f\frac{2.997924580 \times 10^8 \frac{m}{s} + v_P}{2.997924580 \times 10^8 \frac{m}{s} - v_P},$$

where $v_P$ is the velocity of the precipitation which is positive when it is moving towards the radar and negative when it is moving away (2 marks).

13. Calculate the received frequency by the radar for precipitation moving at 10.000000000 km/h towards the radar. The radar emits a pulse of frequency 5.600000000 GHz. Give your answer to 10 significant figures and use c=$2.997924580 \times 10^8$m/s (1 mark).

14. Calculate the received frequency by the radar for precipitation moving at 10.000000000 km/h away from the radar. The radar emits a pulse of frequency 5.600000000 GHz. Give your answer to 10 significant figures and use c=$2.997924580 \times 10^8$m/s (1 mark).




Weather Pattern used in Questions 14-17

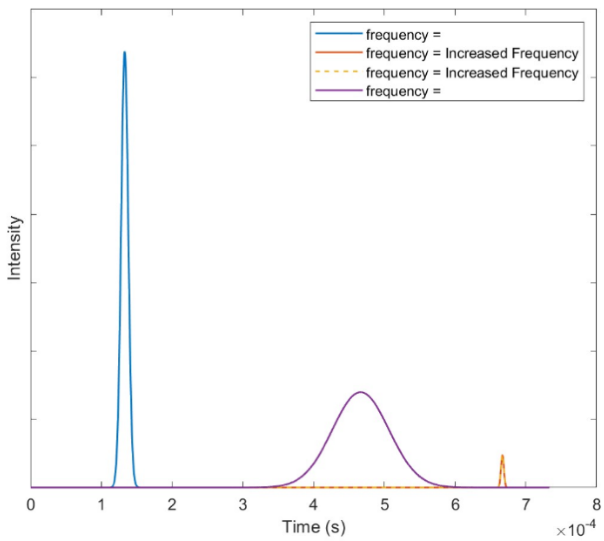
15. For the above weather pattern, explain why there are three peaks in its intensity-time graph (1 mark).

16. What is the received frequency from the pulse reflected by the precipitation 20 km from the radar? Give your answer to 10 significant figures (2 marks).

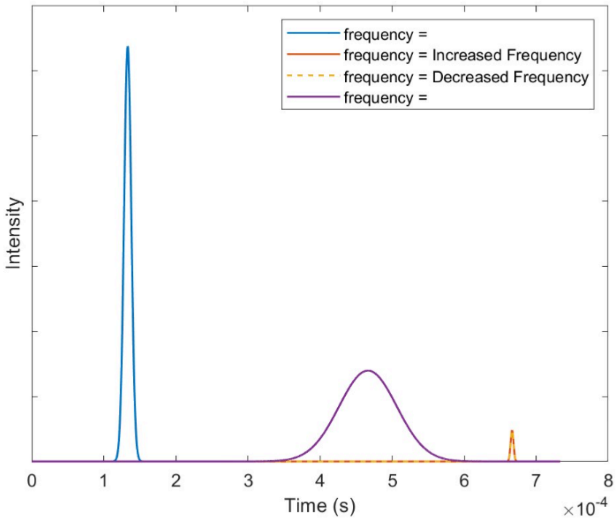
17. What is the received frequency from the pulse reflected by the precipitation 70 km from the radar? Give your answer to 10 significant figures (2 marks).

18. Which of the graphs below (labelled a, b, c and d) could correspond to data received by the radar from the above weather pattern? (1 mark)

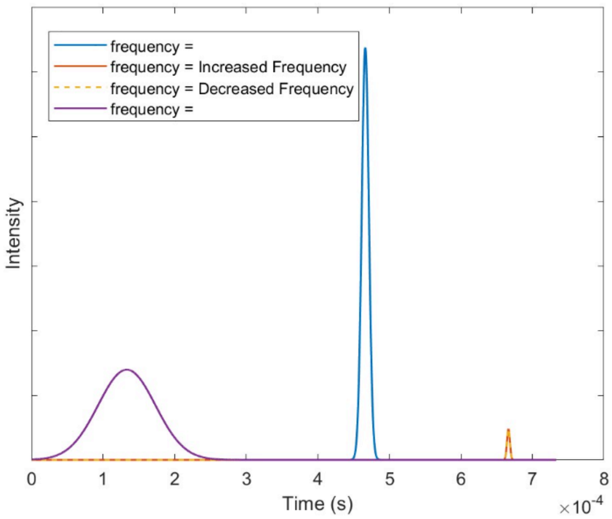
a.



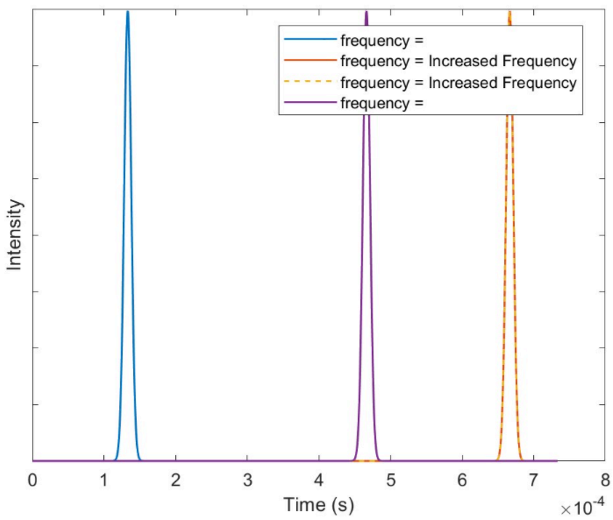
b.


frequency =
frequency = Increased Frequency
frequency = Decreased Frequency
frequency =
Intensity
0
1
2
3
4
5
6
7
8
Time (s)
×10^-4

c.


frequency =
frequency = Increased Frequency
frequency = Decreased Frequency
frequency =
Intensity
0
1
2
3
4
5
6
7
8
Time (s)
×10^-4

d.


frequency =
frequency = Increased Frequency
frequency = Increased Frequency
frequency =
Intensity
0
1
2
3
4
5
6
7
8
Time (s)
×10^-4

**Section C: Bounce-back Beam Biopsy**

**Suggested time: 30 minutes**

Consider two solid, spherical masses, one with mass $m_1$, and one with mass $m_2$. Assuming that $m_2$ is initially at rest, and that $m_1$ is incident on $m_2$ with some energy $E_0$, the particles will scatter with final energies $E_1$ and $E_2$ respectively (as shown).

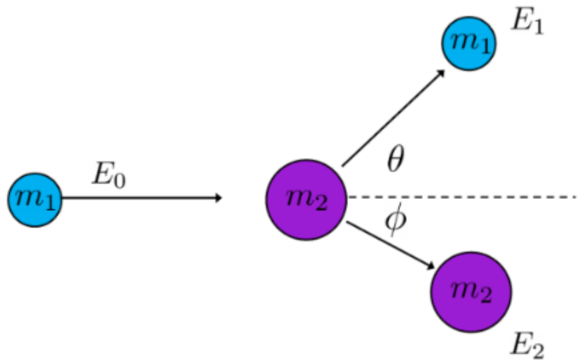



Figure 1: Scattering schematic

1. Write an equation for conservation of energy for this process. Only include the following variables in your answer:
- $m_1$ – the mass of the incoming particle
- $m_2$ – the mass of the originally stationary target
- $E_0$ – the kinetic energy of $m_1$ before the collision
- $E_1$ – the kinetic energy of $m_1$ after the collision
- $E_2$ – the kinetic energy of $m_2$ after the collision

(1 mark)

2. Write two equations for conservation of momentum, accounting for the scattering angles $\phi$ and $\theta$ (4 marks).

3. Solve these equations to find an expression for the final energy of $m_1$, ($E_1$), as a function of initial energy, masses and scattering angle $\theta$. Show that:

$$E_1 = \left(\frac{m_1 \cos\theta \pm \sqrt{m_2^2 - m_1^2 \sin\theta}}{m_1 + m_2}\right)^2 E_0$$

(4 marks)

If you have not solved this in 10 minutes, it is recommended that you move on and just use this result for the later parts of the question.

4. The expression in question 3 contains a "plus-minus" sign ($\pm$). This would seem to imply that we can get two different final energies! Explain under what circumstances we take the positive solution, and when we take the negative, and why ( 4 marks).

5. Using the expression in question 3, take the limits where $m_1 \ll m_2$, and $m_2 \ll m_1$. What do you notice about scattering angles? Can you describe why you see this? (4 marks)
*Hint: are any scattering angles forbidden?*

6. If two particles are now electrically charged, they will experience a repulsive force throughout the process. Does conservation of momentum still hold in this case? Explain why/why not. (2 marks)

Knowledge of an incoming particle's energy and incoming angle, along with the final angle and energy, gives information about what particle scatters off. This technique underpins analysis of samples of solids, where the chemical composition is unknown. Assume for the following questions that the unknown solid contains the following minerals, with their chemical composition listed:

- Diopside $CaMgSi_2O_6$
- Calcite $CaCO_3$
- Dolomite $CaMg(CO_3)_2$
- Pyrite $FeS_2$
- Forsterite $Mg_2SiO_4$

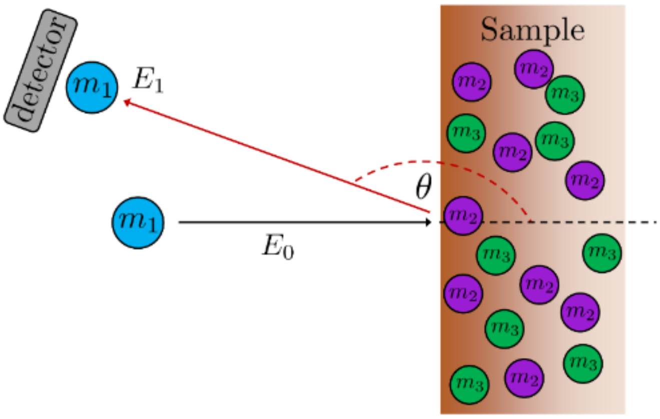



Figure 2: Schematic of an experimental setup which allows investigation of an unknown sample, via bombarding the sample with particles of energy $E_0$ and mass $m_1$. This particular sample has two kinds of elements with masses $m_2$ and $m_3$, but in general, there may be many more.

The list below contains the atomic mass for each of the elements in this question. The unit u stands for relative atomic mass unit, a convenient unit of measurement for the mass of atoms. A periodic table is included at the end of the problem.

7. There is a beam of high-energy silicon ions incident on your sample. Under these conditions, which of the minerals produce an observable signal through backscattering that would allow them to be distinguished from the other minerals? Explain your answer. (5 marks)

8. Instead, consider now that you have an oxygen-16 beam. Is it possible to determine the relative amounts of calcite and dolomite in a sample where they are mixed? Assume that we know the sample only contains these two compounds. (2 marks)

Using the same experimental set-up above, now assume that there is a probing source of $\alpha$ (alpha) particles. This is shown in Figure 3 below. $\alpha$-particles are simply helium-4 nuclei (i.e. no electrons, just the nucleus). These $\alpha$-particles are produced by an Americium-241 source, which then undergoes alpha decay. The alpha decay process produces alpha particles with an energy of 5.486 MeV.

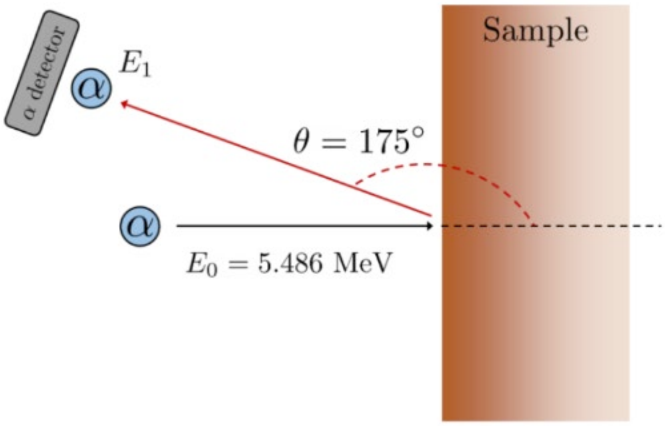



Figure 3: Schematic of experimental set-up, for the $\alpha$-particle scattering.

9. Let's say you have two pieces of elemental, grey metal. One is a thick piece, and the other is thin. You know one is composed of iron, and one is tin, but you've forgotten which is which! Luckily, you have made measurements of the scattered $\alpha$-particles using the experimental apparatus shown in Figure 3. Shown below in Figure 4 are the energy spectra obtained from each sample. The x-axis is the energy of the scattered $\alpha$-particle, measured at an angle $\theta = 175°$. The y-axis is proportional to the number of $\alpha$-particles measured with that energy. Which graph corresponds to which element, and which is the thicker piece? Explain your reasoning with calculation and words. (4 marks)

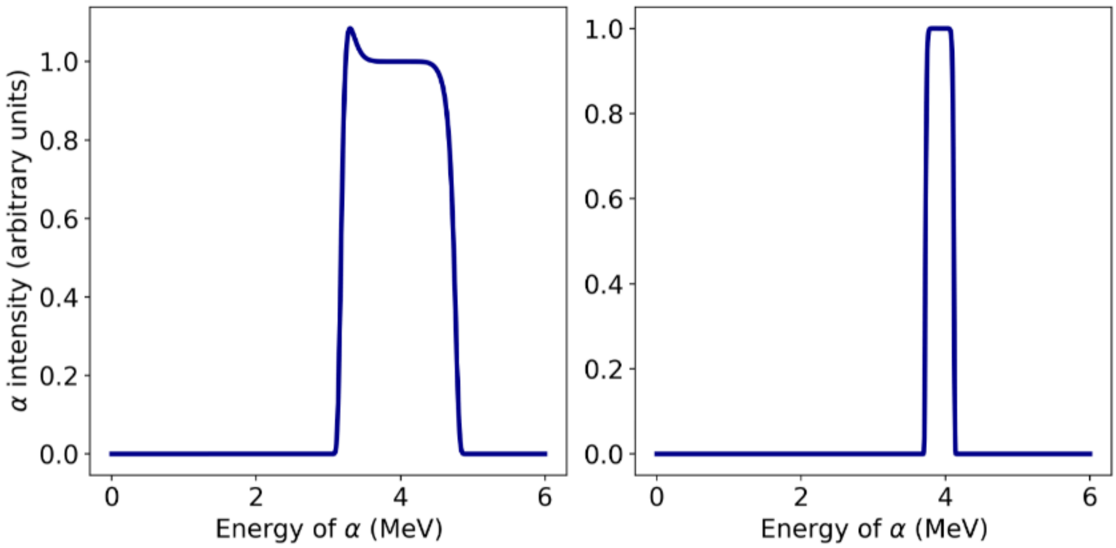



Figure 4: Energy spectra for tin and iron. Note MeV stands for megaelectron volt, a convenient unit for measuring the energy of atoms.

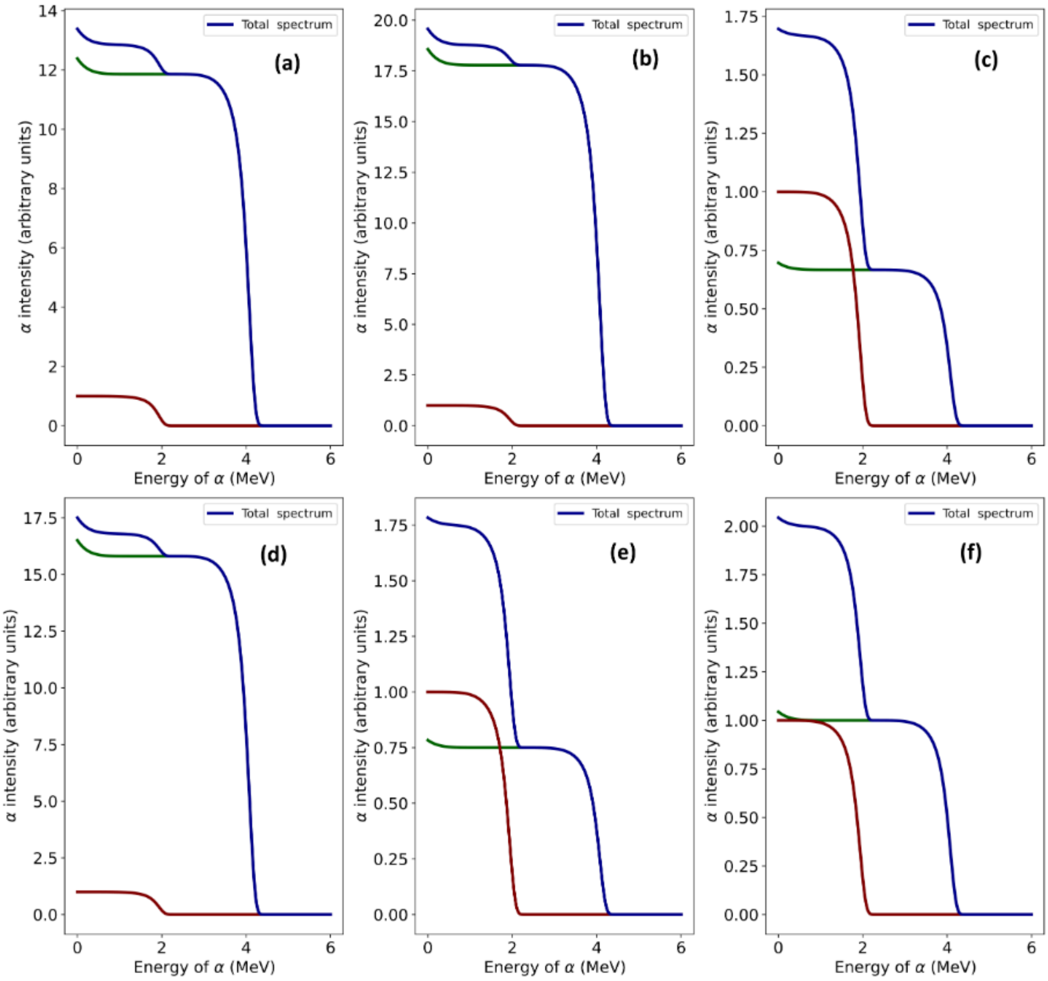
While we can find the energy of the scattered particle for a given angle, this gives no information about the probability of scattering into that angle. Because the $\alpha$-particles are charged (+2), and they scatter off nuclei which are also charged, this changes the probability of scattering into a particular angle. This probability is approximately given by:

$$P(\theta, m_2) \approx \left(\frac{Z_1 Z_2 e^2 (1+\gamma)}{4E_0}\right)^2 \frac{1}{\sin^4(\theta/2)} \frac{(1+\gamma^2+2\gamma\cos\theta)^{3/2}}{1+\gamma\cos\theta}$$

Technically, this is not a probability, but is *proportional* to the probability to scatter into an angle $\theta$. Here, $Z_1$ and $Z_2$ are the charges of the nuclei (proportional to the number of protons in the nuclei), $m_1$ and $m_2$ are the masses of the nuclei, and $\gamma = \frac{m_1}{m_2}$.

10. Assume that we have an unknown compound that has two elements, $X$ and $Y$, in a compound $X_kY_n$, where $k, n$ are integers indicating the ratio of $X$ and $Y$. Furthermore, that elements $X$ and $Y$ have masses $m_X$ $m_Y$, and charges $Z_X$ $Z_Y$ respectively. If the same experimental set-up in Figure 3 is used, write a ratio for the heights of the peaks from $X$ and $Y$, in terms of the above quantities, and $m_\alpha$. (4 marks)

11. This measurement technique allows us to determine what elements are present in a material, and also in what ratio. Consider three common oxides of iron: wüstite (FeO), magnetite ($Fe_3O_4$), and hematite ($Fe_2O_3$). Determine which of the three spectra below correspond to each oxide of iron, explaining your reasoning with words and calculations. Note that in the graphs below, the red and green lines show the contribution of each element towards the total spectrum. For this problem let $\theta = 175°$ and $E_0 = 5.486$ MeV . (6 marks)



# Periodic Table of the Elements

| 1 IA 1A | 2 IIA 2A | 3 IIIB 3B | 4 IVB 4B | 5 VB 5B | 6 VIB 6B | 7 VIIB 7B | 8 | 9 VIII 8 | 10 | 11 IB 1B | 12 IIB 2B | 13 IIIA 3A | 14 IVA 4A | 15 VA 5A | 16 VIA 6A | 17 VIIA 7A | 18 VIIIA 8A |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 **H** Hydrogen 1.008 | | | | | | | | | | | | | | | | | 2 **He** Helium 4.003 |
| 3 **Li** Lithium 6.941 | 4 **Be** Beryllium 9.012 | | | | | | | | | | | 5 **B** Boron 10.811 | 6 **C** Carbon 12.011 | 7 **N** Nitrogen 14.007 | 8 **O** Oxygen 15.999 | 9 **F** Fluorine 18.998 | 10 **Ne** Neon 20.180 |
| 11 **Na** Sodium 22.990 | 12 **Mg** Magnesium 24.305 | | | | | | | | | | | 13 **Al** Aluminum 26.982 | 14 **Si** Silicon 28.086 | 15 **P** Phosphorus 30.974 | 16 **S** Sulfur 32.066 | 17 **Cl** Chlorine 35.453 | 18 **Ar** Argon 39.948 |
| 19 **K** Potassium 39.098 | 20 **Ca** Calcium 40.078 | 21 **Sc** Scandium 44.956 | 22 **Ti** Titanium 47.867 | 23 **V** Vanadium 50.942 | 24 **Cr** Chromium 51.996 | 25 **Mn** Manganese 54.938 | 26 **Fe** Iron 55.845 | 27 **Co** Cobalt 58.933 | 28 **Ni** Nickel 58.693 | 29 **Cu** Copper 63.546 | 30 **Zn** Zinc 65.38 | 31 **Ga** Gallium 69.723 | 32 **Ge** Germanium 72.631 | 33 **As** Arsenic 74.922 | 34 **Se** Selenium 78.971 | 35 **Br** Bromine 79.904 | 36 **Kr** Krypton 83.798 |
| 37 **Rb** Rubidium 85.468 | 38 **Sr** Strontium 87.62 | 39 **Y** Yttrium 88.906 | 40 **Zr** Zirconium 91.224 | 41 **Nb** Niobium 92.906 | 42 **Mo** Molybdenum 95.95 | 43 **Tc** Technetium 98.907 | 44 **Ru** Ruthenium 101.07 | 45 **Rh** Rhodium 102.906 | 46 **Pd** Palladium 106.42 | 47 **Ag** Silver 107.868 | 48 **Cd** Cadmium 112.414 | 49 **In** Indium 114.818 | 50 **Sn** Tin 118.711 | 51 **Sb** Antimony 121.760 | 52 **Te** Tellurium 127.6 | 53 **I** Iodine 126.904 | 54 **Xe** Xenon 131.294 |
| 55 **Cs** Cesium 132.905 | 56 **Ba** Barium 137.328 | 57-71 | 72 **Hf** Hafnium 178.49 | 73 **Ta** Tantalum 180.948 | 74 **W** Tungsten 183.84 | 75 **Re** Rhenium 186.207 | 76 **Os** Osmium 190.23 | 77 **Ir** Iridium 192.217 | 78 **Pt** Platinum 195.085 | 79 **Au** Gold 196.967 | 80 **Hg** Mercury 200.592 | 81 **Tl** Thallium 204.383 | 82 **Pb** Lead 207.2 | 83 **Bi** Bismuth 208.980 | 84 **Po** Polonium [208.982] | 85 **At** Astatine 209.987 | 86 **Rn** Radon 222.018 |
| 87 **Fr** Francium 223.020 | 88 **Ra** Radium 226.025 | 89-103 | 104 **Rf** Rutherfordium [261] | 105 **Db** Dubnium [262] | 106 **Sg** Seaborgium [266] | 107 **Bh** Bohrium [264] | 108 **Hs** Hassium [269] | 109 **Mt** Meitnerium [278] | 110 **Ds** Darmstadtium [281] | 111 **Rg** Roentgenium [280] | 112 **Cn** Copernicium [285] | 113 **Nh** Nihonium [286] | 114 **Fl** Flerovium [289] | 115 **Mc** Moscovium [289] | 116 **Lv** Livermorium [293] | 117 **Ts** Tennessine [294] | 118 **Og** Oganesson [294] |

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| **Lanthanide Series** | 57 **La** Lanthanum 138.905 | 58 **Ce** Cerium 140.116 | 59 **Pr** Praseodymium 140.908 | 60 **Nd** Neodymium 144.243 | 61 **Pm** Promethium 144.913 | 62 **Sm** Samarium 150.36 | 63 **Eu** Europium 151.964 | 64 **Gd** Gadolinium 157.25 | 65 **Tb** Terbium 158.925 | 66 **Dy** Dysprosium 162.500 | 67 **Ho** Holmium 164.930 | 68 **Er** Erbium 167.259 | 69 **Tm** Thulium 168.934 | 70 **Yb** Ytterbium 173.055 | 71 **Lu** Lutetium 174.967 |
| **Actinide Series** | 89 **Ac** Actinium 227.028 | 90 **Th** Thorium 232.038 | 91 **Pa** Protactinium 231.036 | 92 **U** Uranium 238.029 | 93 **Np** Neptunium 237.048 | 94 **Pu** Plutonium 244.064 | 95 **Am** Americium 243.061 | 96 **Cm** Curium 247.070 | 97 **Bk** Berkelium 247.070 | 98 **Cf** Californium 251.080 | 99 **Es** Einsteinium [254] | 100 **Fm** Fermium 257.095 | 101 **Md** Mendelevium 258.1 | 102 **No** Nobelium 259.101 | 103 **Lr** Lawrencium [262] |

## Section D: Seeing Secchis

## Suggested time: 30 minutes

A Secchi disk is a tool used to measure the turbidity (cloudiness) of a body of water. It consists of a disk split into quarters alternatingly painted black and white. The disk is lowered into the water until the ’secchi’ depth at which it can no longer be seen. 'Attenuation' is a measure of how light decreases with depth in the water. In this question students would need to determine the factors affecting the secchi depth and methods to calculate the attenuation coefficient of the water from the secchi depth. These factors will include the light level and contrast level the black and white.

This is a Secchi disk:

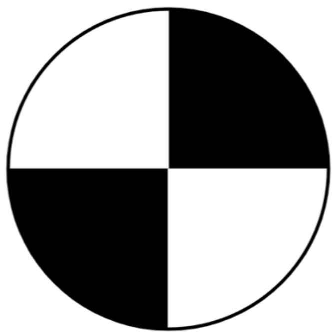


Figure 1: A Secchi Disk

If you lower it deeper and deeper underwater it will get to a point where you can no longer distinguish between the white and black sections. Why? Because light ‘attenuates’ through water. Attenuation is a measure of how light intensity decreases as it passes through a medium. As the disk gets deeper, so much light will be lost through the water that we can no longer distinguish the black and white disks. This process occurs at relatively shallow depths due to the ambient light, in this case light reflected off the water.

Attenuation of light can be calculated using the Beer-Lambert Law:

$$I = I_0 e^{-\mu z},$$

where:

- $\mu$ is the attenuation coefficient of the fluid, a measure of how quickly the light level decreases. $\mu$ is a measure of turbidity.
- $e$ is the Euler number = 2.71828
- $z$ is the path length of the light through the fluid
- $I$ is the intensity of the light observed once the light passes through a path length of fluid $z$
- $I_0$ is the initial intensity of the light source.

However, if there is an ambient light intensity, $I_f$, then the light intensity will not decay to zero but instead to the ambient level as shown in the equation below:

$$I = (I_0 - I_f)e^{-\mu z} + I_f$$

Secchi Disk in Water Tank

Ambient light reflected off cloudy water has nonzero intensity

1. If a Secchi disk is a depth $d$ below the surface of the water, write an expression for the total path length $z$ for light that passes straight down from the surface to the disk and the reflects and comes back to the observer at the surface. (1 mark)

2. Assuming that the initial light intensity is greater than the ambient light intensity, describe what happens to the intensity of light with depth. What happens to the light intensity with depth if the initial light intensity is equal to the ambient light intensity? (2 marks)

Note that from this point onward in this problem, we assume that the ambient light is caused by reflection off the surface of the fluid. Therefore $I_f < I_0$. Note that in reality, light scatters from each infinitesimal layer of water, however the surface reflection can be a good enough approximation for this scenario. When light hits a surface a fraction of the light reflects according to the following equation:

$$I_r = RI_i,$$

where:

- $R$ is the reflectivity of the surface. For the Secchi disk, the white material will have reflectivity $R_w \approx 1$, and the black will have reflectivity $R_k \approx 0$. The reflectivity of the water surface is $R_f$.
- $I_r$ is the intensity of the light reflected from the surface
- $I_i$ is the intensity of the light that lands on the surface (incident on the surface)

The surface of the water will also reflect some of the initial light. This creates the ambient light intensity given by $I_f = R_f I_0$. Hence the intensity formula becomes:

$$\frac{I}{I_0} = \left(1 - R_f\right)e^{-\mu z} + R_f \ .$$

Consider the diagram of the Secchi disk in action in Figure 2 below:

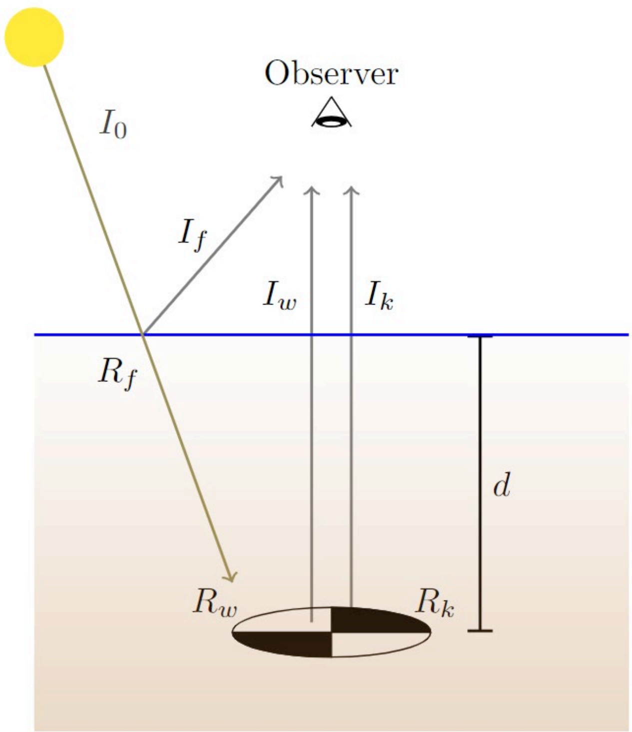



Figure 2: Secchi Disk in use.

We say that the Secchi depth is the depth at which the white and black segments can no longer be distinguished. This happens when their intensities at the observer become sufficiently similar.
The contrast threshold, $C_T$, is a measure of how similar two light intensities can be before they can no longer be distinguished. The disk can no longer be distinguished when the following relationship holds:

$$\frac{I_w - I_k}{I_k} < C_T,$$

where:

- $I_w$ is the intensity of light observed when looking at the white part of the Secchi Disk, and
- $I_k$ is the intensity of light observed when looking at the black part of the Secchi Disk.

3. Explain why the intensity of light reflected off the white part of the Secchi Disk that reaches the observer at the surface is given by the following:
$$\frac{I_w}{I_0} = (1 - R_f)R_w e^{-\mu z} + R_f\ .$$
Ensure you explain where each of the terms come from. Does this formula account for ambient light reflected off the water? (3 marks)

4. Determine an expression for the intensity of light reflected off the black part of the Secchi disk that reaches the observer at the surface. Ensure your formula also includes a term for the ambient light. (1 mark)

5. The contrast is given by the following formula: $C = \frac{I_w - I_k}{I_k}$.
Using your expressions above, write a simplified expression for the contrast in terms of $R_w, R_k, R_f, \mu$ and $z$. (2 marks)

6. Solve your equation from the previous question to derive the following expression for $\mu$. Show your working. (3 marks)

$$\mu = \frac{1}{z}\ln\left((1 - R_f)\left(\frac{R_w - R_k}{R_f C_a} - \frac{R_k}{R_f}\right)\right)$$

7. Assume that the fluid has a reflectance of $R_f = 0.2$ and that the disk remains discernible while $C \geq C_t = 0.01$. Assume $R_k = 0.1$ and $R_w = 0.9$. If the lake has a depth of 4 m, what is the minimum turbidity ($\mu$) that can be measured? (1 mark)

8. Which of the following changes would allow the measurement of a smaller minimum turbidity in thus lake? Explain your reasoning. (3 marks)

    a. Using a specialised detector with $C_t = 0.001$ instead of your eye,
    b. Increasing $R_w$ ,
    c. Increasing $R_k$ ,
    d. Waiting until the sun is at the highest point in the sky to perform the measurement,
    e. Using a larger Secchi Disk.

9. The reflectance of the fluid is only known to $\pm 20\%$. Will the upper ($+20\%$) or lower ($-20\%$) value lead to more uncertainty in the final calculated turbidity? (1 mark)

10. Find extremal turbidity bound given your answer to the previous question, and thus calculate the uncertainty to which the turbidity is measured. (1 mark)

## Section E: Trailer-Towing Truck

**Suggested time: 15 minutes**

A truck is towing a trailer with an unusual towing device: a light spring with spring constant $k = 200$ N.m$^{-1}$ and natural length $\ell_0 = 1.00$ m, as shown in Figure 1. The truck is travelling at the speed limit (a constant speed of $v = 10.0$ m.s$^{-1}$ relative to the ground). The road is on a hill such that the truck first travels on flat, horizontal ground, then up a $\theta = 20°$ incline relative to the horizontal, and then back to the horizontal ground as shown in Figure 2. Assume that the truck and trailer smoothly transition between the different inclines, and each segment is sufficiently long that the system reaches equilibrium. The mass of the trailer is $m$=500 kg and the friction coefficient between the wheels and ground is $\mu$=0.05. Assume the gravitational acceleration $g$=9.80 m.s$^{-1}$.

Figure 1: Truck connected to trailer with a spring

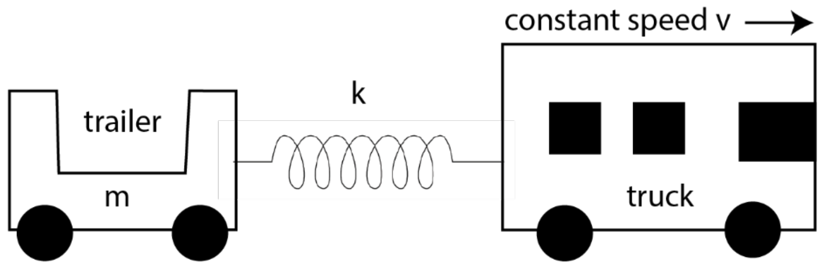
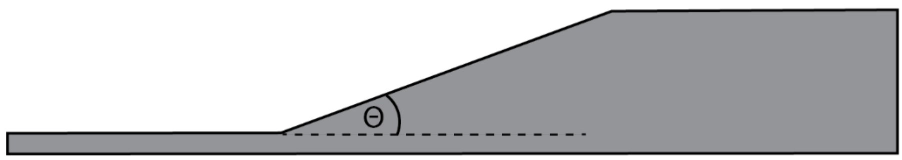
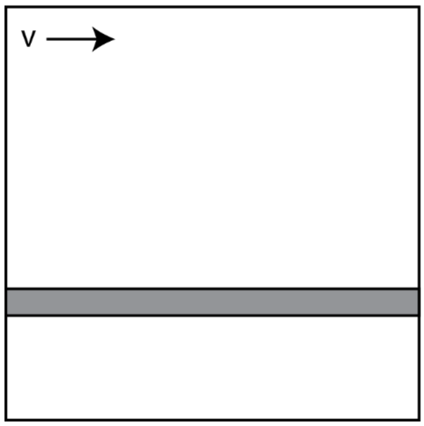


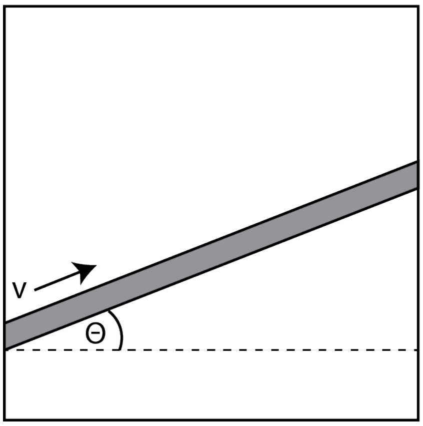

Figure 2: Hill Geometry




1. Draw a force diagram of the forces on the trailer while on horizontal ground. (1 mark)



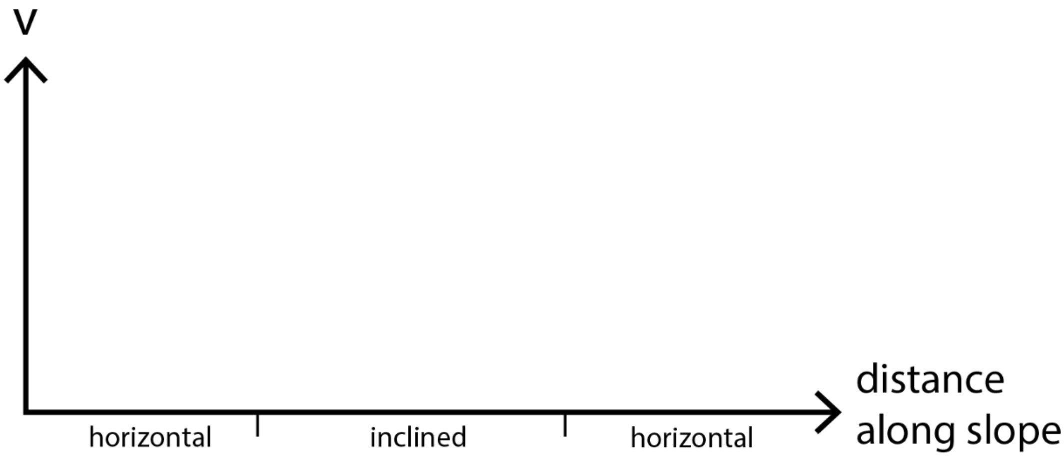
2. Draw a force diagram of the forces on the trailer while on an incline. (1 mark)




3. Determine the length of the spring while the trailer is being towed on flat ground, once the system has reached equilibrium. (2 marks)

4. Determine the length of the spring while the trailer is being towed on the inclined slope, once the system has reached equilibrium. (3 marks)

5. Sketch the velocity of the trailer relative to the ground, $v$, as a function of position along the slope using the given axes. (2 marks)




**END OF EXAM**