

*Time Allowed:*

*Reading Time: 10 minutes*

*Examination Time: 120 minutes*

### INSTRUCTIONS

- Attempt ALL questions in both sections of this paper.
- Permitted materials: a *non-programmable, non-graphical* calculator, blue and black pens, lead pencils, an eraser, and a ruler.
- Answer SECTION A on the MULTIPLE CHOICE ANSWER SHEET provided.
- Answer SECTION B in the answer booklet provided. Write your answers to each question on the pages indicated. If you need additional space use the spare pages at the back of the booklet. Write in pen and use pencil only for diagrams and graphs.
- You may attempt the questions in Section B in any order. Make sure that you label which parts are for which questions.
- **Do not write on this question paper. It will not be marked.**
- Do **not** staple the multiple choice answer sheet or the writing booklet to anything. They must be returned as they are.
- Ensure that your diagrams are clear and labelled.
- All numerical answers must have correct units.
- Marks will not be deducted for incorrect answers.

### MARKS

|           |                              |                 |
|-----------|------------------------------|-----------------|
| Section A | 10 multiple choice questions | 10 marks        |
| Section B | 4 written answer questions   | 50 marks        |
|           |                              | <b>60 marks</b> |

**SECTION A: MULTIPLE CHOICE**  
**USE THE ANSWER SHEET PROVIDED**

Throughout, take the acceleration due to gravity to be  $9.8 \text{ ms}^{-2}$ .

**Question 1**

A cricketer leaps into the air and catches a ball while both are in mid air. Which of the cricketer or the ball undergoes the smaller change in momentum?

- a. The cricketer does.
- b. The ball does.
- c. The change in momentum is the same for both the cricketer and the ball.
- d. You can't tell without knowing the final velocity of the combined cricketer-ball mass.
- e. The result depends on the energy absorbed by the cricketer's hands while catching the ball.

**Question 2**

Just before the cricketer caught the ball her kinetic energy was the same as the ball's kinetic energy. Just before the cricketer catches the ball, which of the following statements is true?

- a. The ball has a greater speed than the cricketer.
- b. The cricketer has a greater speed than the ball.
- c. The cricketer and the ball have the same speed.
- d. The kinetic energy cannot give information about their speeds.
- e. The directions of the cricketer and the ball must be taken into account to compare their speeds.

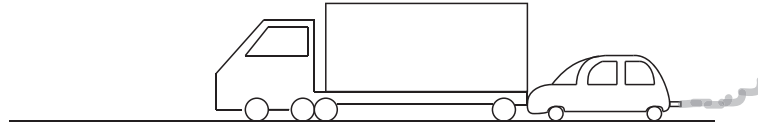
**Question 3**

Tara likes playing with bubbles and watching the bright colours in them change. She notices that bubbles which have dark patches pop sooner than those with no dark patches. She concludes that dark patches cause bubbles to pop. Which of the following is the best statement of the flaw in Tara's argument?

- a. It ignores the effects of bubbles colliding with other objects.
- b. It assumes that because dark patches occur just before popping, dark spots cause popping.
- c. It ignores the fact that some bubbles pop when there are no dark patches.
- d. It ignores the effects of other colours on whether bubbles pop.
- e. There is no flaw in Tara's argument. Dark patches cause popping.

#### Question 4

A large truck breaks down on the freeway and receives a push to the nearest exit by a small car as shown below.



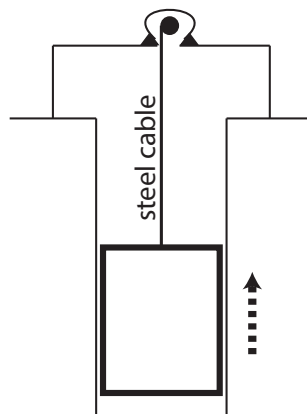
While the car, still pushing the truck, is speeding up to get to cruising speed

- the amount of force with which the car pushes on the truck is equal to that with which the truck pushes back on the car.
- the amount of force with which the car pushes on the truck is smaller than that with which the truck pushes back on the car.
- the amount of force with which the car pushes on the truck is greater than that with which the truck pushes back on the car.
- the car's engine is running so the car pushes against the truck, but the truck's engine is not running so the truck cannot push back against the car. The truck is pushed forward simply because it is in the way of the car.
- neither the car nor the truck exert any force on the other. The truck is pushed forward simply because it is in the way of the car.

#### Question 5

An elevator is being lifted up an elevator shaft at a constant speed by a steel cable as shown in the figure below. All frictional effects are negligible. In this situation, the forces on the elevator are such that:

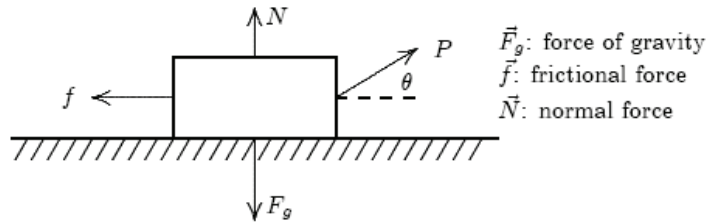
- the upwards force of the cable is greater than the downward force of gravity.
- the upward force of the cable is equal to the downward force of gravity.
- the upward force of the cable is smaller than the downward force of gravity.
- the upward force of the cable is greater than the sum of the downward force of gravity and a downward force due to air.
- none of the above. (The elevator goes up because the cable is being shortened, not because an upwards force is exerted on the elevator by the cable.)



Elevator going up at constant speed

**Question 6**

A donkey pulls a wooden box along rough flat ground at a constant speed by means of a force  $\vec{P}$  (magnitude  $P$ ) as shown. In the diagram,  $f$  is the magnitude of the frictional force,  $N$  is the magnitude of the normal force, and  $F_g$  is the magnitude of the force of gravity. Which of the following options must be true?



- a.  $P = f$  and  $N = F_g$
- b.  $P = f$  and  $N > F_g$
- c.  $P > f$  and  $N < F_g$
- d.  $P > f$  and  $N = F_g$
- e. none of these.

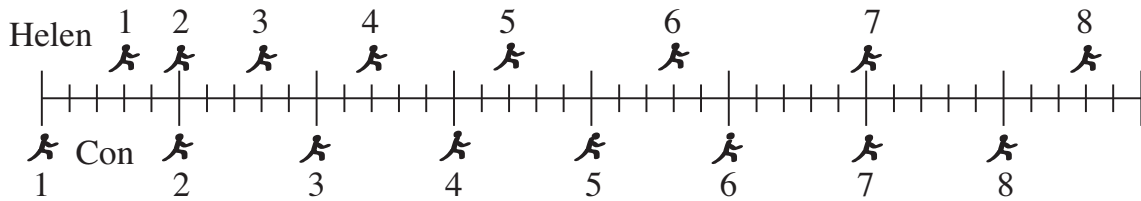
**Question 7**

The donkey of Question 6 now pulls harder and doubles the magnitude of the force  $P$ . Which of the following options must be true?

- a.  $F_g$  and  $N$  remain constant but  $f$  increases to counter the increase in  $P$ .
- b.  $F_g$ ,  $N$  and  $f$  all remain constant despite the increase in  $P$ .
- c.  $F_g$  and  $f$  remain constant but  $N$  decreases due to the increase in  $P$ .
- d.  $F_g$  remains constant but  $N$  and hence  $f$  decrease due to the increase in  $P$ .
- e. none of these.

### Question 8

The positions of two runners, Helen and Con, are shown below. The runners are shown at successive 0.20 second intervals, and they are moving towards the right.

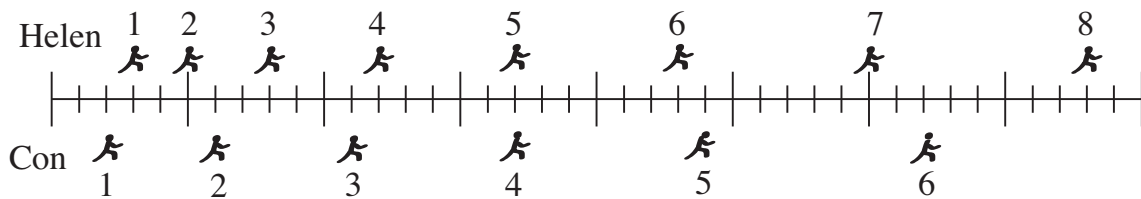


Do Helen and Con ever have the same speed?

- a. No.
- b. Yes, at instant 2.
- c. Yes, at instant 7.
- d. Yes, at instants 2 and 7.
- e. Yes, at some time during the interval 4 to 5.

### Question 9

The positions of two runners, Helen and Con, are shown below. The runners are shown at successive 0.20 second intervals, and they are moving towards the right.



Which of the following statements best describes how the accelerations of the runners are related.

- a. The acceleration of Con is greater than the acceleration of Helen.
- b. The acceleration of Helen is greater than the acceleration of Con.
- c. The accelerations of Helen and Con are equal. Both accelerations are equal to zero.
- d. The accelerations of Helen and Con are equal. Both accelerations are greater than zero.
- e. Not enough information is given to answer the question.

### Question 10

Which of the following is the best estimate of the volume of an orange.

- a.  $3 \times 10^{-5} \text{ m}^3$
- b.  $3 \times 10^{-4} \text{ m}^3$
- c.  $3 \times 10^{-3} \text{ m}^3$
- d.  $3 \times 10^{-2} \text{ m}^3$
- e.  $3 \times 10^{-1} \text{ m}^3$

## SECTION B: WRITTEN ANSWER QUESTIONS

### USE THE ANSWER BOOKLET PROVIDED

**Note:** Suggested times are given for section B as a general guide only. You may take more or less time on any question – everyone is different.

#### Question 11

*Suggested Time: 35 min*

Maria is a connoisseur of fine teas. She is preparing her new tea with boiling water in her cylindrical mug with inner radius  $r = 4$  cm and height  $h = 10$  cm. Her mug has a lid for better insulation.

In a short time  $\Delta t$  the amount of heat which flows out of the tea is given by

$$\Delta Q = \frac{\kappa S (T_{\text{tea}} - T_{\text{room}})}{d} \Delta t \quad , \quad (1)$$

where  $S$  is the inner surface area of the mug and  $d = 5$  mm is the thickness of the mug walls.

As heat flows from the tea the change in its temperature  $\Delta T$  given by

$$\Delta Q = m \cdot c \cdot \Delta T \quad , \quad (2)$$

where  $m$  is the mass of the water.

#### Data

Temperature of boiling water:  $T_{\text{boil}} = 100$  °C

Thermal conductivity of mug:  $\kappa = 1.0$  W m<sup>-1</sup> K<sup>-1</sup>

Room temperature:  $T_{\text{room}} = 25$  °C

Density of water:  $\rho = 1000$  kg m<sup>-3</sup>

Specific heat capacity of water:  $c = 4180$  J kg<sup>-1</sup> K<sup>-1</sup>

Maria pours her tea, filling her mug completely, and immediately places the lid on her mug.

- a)
  - (i) Find the temperature  $T_{30}$  of Maria's tea after 30 seconds have elapsed and fill in the first line of the table on p. 2 of the Answer Booklet.
  - (ii) Complete the rest of the table on p. 2 of the Answer Booklet.  
Hint: use your answers from one line to help you with the next line.
  - (iii) Plot a graph of  $T_{\text{tea}}$  vs. time on p. 3 of the Answer Booklet. Include times from 0 s up to and including 5 min.
- b) Find the time it takes Maria's tea to reach 55 °C.
- c) Maria decides to reheat her tea from 55 °C to 70 °C with a small 1.1 kW heater she places into the tea. How long will it take the tea to reach 70 °C? You may neglect the heat flow out of the tea.
- d) Estimate the size of the error in your answer to part (c) due to neglecting heat flow out of the tea. Use your estimate to comment on whether your answer to part (c) is reasonable.

### Question 12

*Suggested Time: 30 min*

A block of mass  $m$  sits against an unextended spring with spring constant  $k_1$  on a frictionless surface as shown below. A short distance beyond the mass, there is a vertical cliff of height  $h$  that drops off to a rough surface with kinetic friction coefficient  $\mu_k$ .



When compressed or extended, the spring exerts a restoring force

$$F = -kx \quad ,$$

where  $k$  is the spring constant and  $x$  is the change in length of the spring. The elastic potential energy stored in the spring during this deformation is given by

$$U = \frac{1}{2}kx^2 \quad .$$

The block is pushed against the spring with displacement  $x_1$  and then released at time  $t = 0$ .

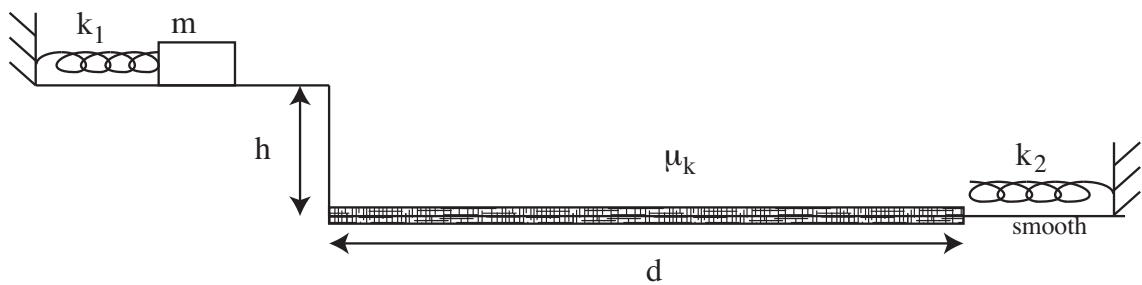
- What is the velocity of the block immediately after it loses contact with the spring?
- After reaching the cliff, the block continues to move through the air until it hits the rough surface below. Once it hits the rough surface, a frictional force acts against the horizontal motion of the block.

On the axes given on p. 4 of the Answer Booklet sketch as functions of time, starting when the block is released and ending when the block is at rest again, the

- kinetic energy,
- gravitational potential energy, and
- elastic potential energy.

**Use the same axes for all three sketches and label each line clearly.**

- c) A section of the rough ground a distance  $d$  from the cliff is made smooth (frictionless) and a second spring, with constant  $k_2 = 2k_1$  is placed on it, as shown below.



- (i) On the axes on p. 5 of the Answer Booklet, sketch the maximum compression of the new spring as a function of the maximum compression of the launching spring. Only consider compressions causing the block to land *before* the second spring.
- (ii) Find an expression for the largest possible maximum compression of the second spring that you marked on your sketch for part (c)(i).



### Question 13

*Suggested Time: 18 min*

A lolly factory produces shiny delicious spherical chocolates of radius  $r$ . It packs the chocolates in layers arranged as shown in Figure 1; each layer is stacked directly on top of the one below.

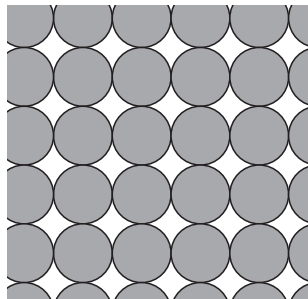


Figure 1: Neatly packed chocolates.

The packing fraction is defined to be the ratio of the volume of some objects divided by the total volume of space which they occupy.

When arranged randomly the chocolates have a packing fraction of 0.64.

- Find the packing fraction of the chocolates when they are neatly packed in stacked layers with the arrangement shown in Figure 1.
- Do the same number of chocolates occupy more space when neatly packed as in Figure 1 or when randomly arranged?

To sell the chocolates the factory packs them into bags which are filled through funnels like that shown in Figure 2. As the bags are filled chocolates are added into the top of the funnel at an average rate of 5 chocolates per second and the level in of chocolates in the top of the funnel is steady.

- How many bags of 30 can be filled per minute through each funnel?
  - How fast do the chocolates shoot out of the funnel into a bag?
  - What is the average downwards speed of the chocolates in the top of the funnel?

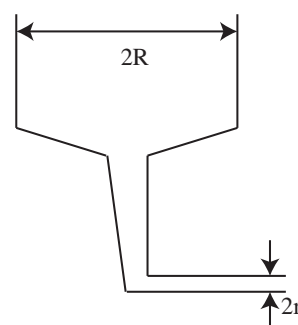


Figure 2: Funnel for filling bags.

### Question 14

*Suggested Time: 18 min*

Astrophysical systems such as galaxies (which contain large numbers of stars) are generally very far from the Earth and solar system, and are moving away from us with some relative velocity  $v$ . Individual stars can be moving with respect to the galaxy; the velocity of this motion is known as the peculiar velocity. Figure 3 shows a galaxy which is moving away from us as a whole; stars in this galaxy orbit its centre and have peculiar velocities.

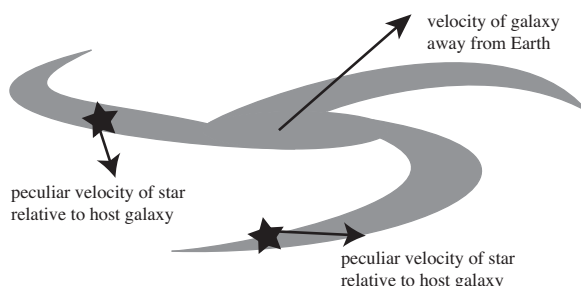


Figure 3: Peculiar velocities of a stars in a galaxy.

The energy from the peculiar motion within a galaxy, rather than its overall movement, is interesting. One way to measure the typical peculiar speed of a star, is to look at the spread in velocities. The standard deviation of the peculiar speeds is called the *velocity dispersion*,  $\sigma$ , and it becomes an estimate of the typical peculiar speed of a star.

- a) The virial theorem applied to an astrophysical system with very large mass  $M$  gives that

$$v^2 = \frac{GM}{R} ,$$

where  $v$  is the average speed of an object orbiting at a radius  $R$ , and  $G$  is the universal gravitational constant. Use the virial theorem to estimate the mass  $M$  of a galaxy, where a population of stars at radius  $R$  from the centre of the galaxy are observed to have velocity dispersion  $\sigma$ .

When the source of a wave is approaching or retreating from an observer the wavelength changes. The size of this change is proportional to the speed of the source. This is known as the Doppler effect and is useful in astrophysics because elements emit light within narrow wavelength regions meaning peaks are observed in objects' spectra.

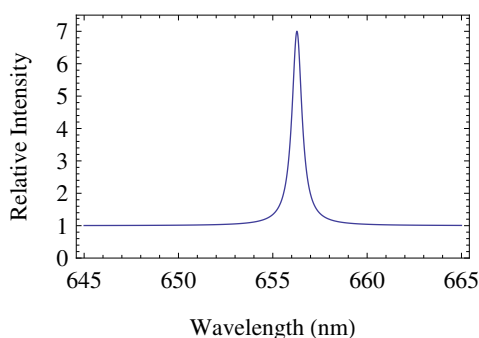


Figure 4: Hydrogen alpha emission spectrum.

- b) (i) Explain, with sketches, how the Doppler effect causes broadening of the Hydrogen alpha emission lines in the fast-rotating broad-line regions of galaxies. On the diagram on p. 8 of the Answer Booklet, draw what the Hydrogen alpha peak in Figure 4 would look like if its source is gas in the broad-line region of a galaxy.
- (ii) Hence, explain how observations of spectra can be used to measure the velocity dispersion of stars in a galaxy.

Most galaxies have supermassive black holes at their centres, which can be a billion times more massive than the Sun. Sometimes a violent event in the supermassive black hole causes changes throughout the fast-rotating region around the centre, called the broad-line region; these changes appear some time later. A technique called reverberation mapping uses the time it takes light to travel across the broad-line region to estimate its radius. The light travel time  $\tau$  is the measured time between observing changes in the front and in the back of the region. The speed of light is  $c$  and the velocity dispersion of the broad-line region is  $\sigma_{\text{BLR}}$ .

- c) (i) Find an expression for the radius  $R_{\text{BLR}}$  of the broad-line region.
- (ii) Estimate the mass  $M_{\text{BH}}$  of a supermassive black hole using reverberation mapping and the virial theorem. You may assume that supermassive black holes are so massive that they account for most of the mass in the centre of a galaxy.

***Integrity of Competition***

*If there is evidence of collusion or other academic dishonesty, students will be disqualified. Markers' decisions are final.*