AUSTRALIAN SCIENCE OLYMPIADS

PHYSICS

2009 NQE Solutions

Time Allowed: 135 minutes
Hint: It's a good idea to read through the paper first!

Comments

- In these solutions, the questions are presented with the solutions directly following each part of each question. At the end of each question, comments from the marker of that question are provided to give additional information about common responses.
- These solutions are a guide only. They provide a sketch of the solutions but not all of the intermediate steps in working that a student would be expected to submit. They do, however, contain enough information that a student should be able to follow one method of solving each question. Students who submit solutions that are valid but do not match what is written here are given full credit, and these solutions should not be read as a statement of the only acceptable answers.

MARKS

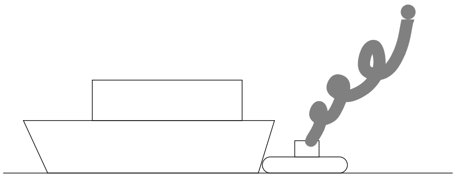
Section A	10 multiple choice questions	10 marks
Section B	5 written answer questions	60 marks
		70 marks

Note: 1 bonus mark (included in the 60) was awarded at the markers' discretion for exceptional insight in any question in Section B.

Section A

Multiple Choice — 1 mark each

Question 1



A large container ship is coming in to port and must be pushed by a tugboat. While the tug and the container ship being pushed are speeding up to cruising speed,

(A) the amount of force with which the tug pushes against the container ship is equal to the amount of force with which the container ship pushes back against the tug.

(B) the amount of force with which the tug pushes against the container ship is smaller than the amount of force with which the container ship pushes back against the tug.

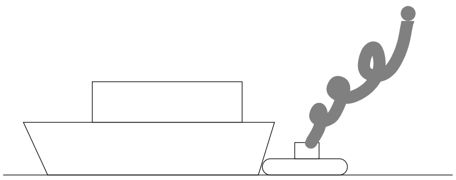
(C) the amount of force with which the tug pushes against the container ship is greater than the amount of force with which the container ship pushes back against the tug.

(D) the tug's engine is running so the tug pushes against the container ship, but the container ship's engine is not running so it can't push back against the tug.

(E) neither the tug nor the container ship exert any force on the other. The container ship is pushed forward simply because it is in the way of the tug.

Solution: (a) – the forces are an action-reaction pair.

Question 2



A large container ship is coming in to port and must be pushed by a tugboat. After the tug and the container ship being pushed have reached a constant cruising speed,

(A) the amount of force with which the tug pushes against the container ship is equal to the amount of force with which the container ship pushes back against the tug.

(B) the amount of force with which the tug pushes against the container ship is smaller than the amount of force with which the container ship pushes back against the tug.

(C) the amount of force with which the tug pushes against the container ship is greater than the amount of force with which the container ship pushes back against the tug.

(D) the tug's engine is running so the tug pushes against the container ship, but the container ship's engine is not running so it can't push back against the tug.

(E) neither the tug nor the container ship exert any force on the other. The container ship is pushed forward simply because it is in the way of the tug.

Solution: (a) – the forces are an action-reaction pair.

If $y = mx + b$ then a plot of y versus x is a straight line with slope m. For the next two questions, consider the equation

$$l + l_0 = \frac{2n-1}{4}\frac{v_s}{f} \quad .$$

Question 3

What is the slope of a plot of $\frac{1}{f}$ versus l? Assume that the other quantities in the equation above are constant.

(A) $\frac{2n-1}{4}v_s$

(B) $\frac{l_0}{v_s}\frac{4}{2n-1}$

(C) The plot will not be a straight line.

(D) $\frac{4}{2n-1}\frac{1}{v_s}$

(E) $\frac{2n-1}{4}(1-l_0)v_s$

Solution: (d) –

$$\frac{1}{f} = \frac{4}{2n-1}\frac{1}{v_s}(l + l_0)$$

Question 4

What is the slope of a plot of f versus $\frac{1}{l}$? Assume that the other quantities in the equation above are constant.

(A) $\frac{2n-1}{4}v_s$

(B) $\frac{l_0}{v_s}\frac{4}{2n-1}$

(C) The plot will not be a straight line.

(D) $\frac{4}{2n-1}\frac{1}{v_s}$

(E) $\frac{2n-1}{4}(1-l_0)v_s$

Solution: (c) –

$$f = \frac{2n-1}{4}v_s\frac{1}{l+l_0} \quad ,$$

so f doesn't depend linearly on $1/l$.

Question 5

A large chicken catches a lightweight paper plane while both are in midair. Which of the chicken and the paper plane undergoes the smaller change in momentum?

(A) The chicken does.

(B) The paper plane does.

(C) The change in momentum is the same for both the chicken and the paper plane.

(D) You can't tell without knowing the final velocity of the combined chicken-plane mass.

(E) The result depends on the energy absorbed by the crumpling of the paper plane in the chicken's beak.

Solution: (c) – momentum is conserved so the magnitudes of the changes in momentum are the same.

Question 6

Someone analysing an electric circuit obtains the equations

$$\begin{aligned} 10\,\mathrm{V} - I_1(15\,\Omega) - I_3(5\,\Omega) &= 0 \ , \\ 2\,\mathrm{V} - I_1(5\,\Omega) - I_3(7\,\Omega) &= 0 \ , \\ I_1 + I_3 &= I_2 \ . \end{aligned}$$

Note that V is the unit volts, and Ω is the unit Ohms. How do the **magnitudes** of the three currents I_1, I_2 and I_3 compare?

(A) $|I_3| > |I_1| > |I_2|$

(B) $|I_2| > |I_3| > |I_1|$

(C) $|I_2| > |I_1| > |I_3|$

(D) $|I_1| > |I_3| > |I_2|$

(E) $|I_1| > |I_2| > |I_3|$

Solution: (e) – combining the first two equations by subtracting 5 times the second from the first gives

$$I_1(10\,\Omega) + I_3(30\,\Omega) = 0$$

so $I_1 = -3I_3$. Substituting into the third equation gives $I_2 = -2I_3$.

Question 7

If you look through a piece of red-tinted glass, everything is seen in shades of red. Similarly, if you look through a piece of blue-tinted glass, everything will be seen in shades of blue. Consider the following statements:

(I) The tinting process makes the glass absorb the corresponding colour, i.e. red-tinted glass strongly absorbs red light, making everything appear red.

(II) The tinting process makes the glass absorb all colours except the corresponding colour, i.e. red-tinted glass will strongly absorb blue and green, but not red.

(III) If you stack the red and blue tinted pieces of glass and look through them, everything will look quite dark.

Which of these statements is/are true?

(A) I and II

(B) I and III

(C) II and III

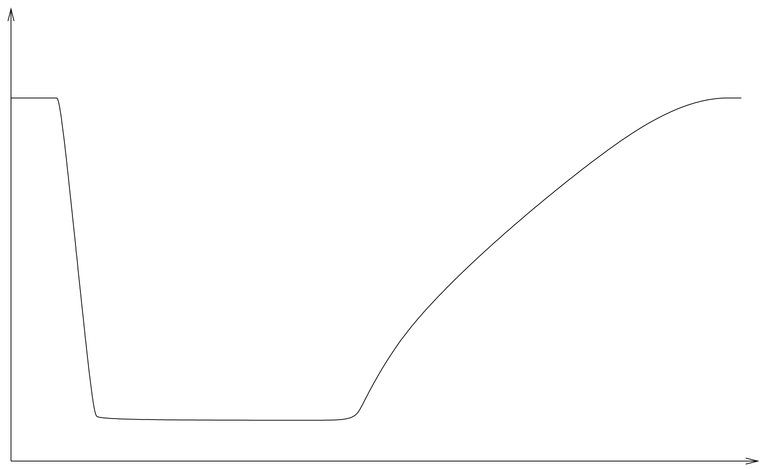
(D) I only

(E) II only

Solution: (c) – (I) is false and (II) is true as for something to appear red, your eyes must receive red light so it must pass through a red-tinted glass. (III) is also true as the red-tinted glass will absorb any blue light that passes through the blue-tinted piece or vice versa so almost no light will pass through.

Question 8

A teacher collects science projects from a class and finds that one page has fallen out. All that it contains is this plot without any axis labels or scales.



Since the plot was unlabeled the teacher has to ask the class if anyone thinks it is theirs to work out whose assignment it came from. Even though they won't get many marks for an unlabeled plot five students claim it as their work, making the following statements. To whom does it belong?

(A) I plotted the number of birds on the island versus time over many years. The number was constant for the first few years but then decreased greatly over a single year because of a disease. After that the number increased more slowly until eventually it was greater than it had originally been.

(B) I plotted the height of a swing above the ground against horizontal distance.

(C) I plotted the volume of water in a jug as I poured out a glass of water, then stopped pouring, then poured another and so on until I had poured four glasses and then filled up the jug from the tap so it had enough water for another four glasses.

(D) I added different masses of compound A to some of compound B so the total initial mass was fixed and measured how much of A was left after they reacted. I plotted the amount of A left against the amount of A I added. When I only added a little A there wasn't much left at the end, but as I added more the amount left first increased and then decreased before increasing again.

(E) I plotted the temperature of my water and ice mixture versus time. First I had room temperature water, then I added ice and stirred until the mixture reached 0 °C. It stayed at 0 °C until the ice had all melted and then slowly increased back up to room temperature.

Solution: (e) – the key features are that the temperature was constant, then decreased and remained constant at a lower value before increasing back to the initial value.

Question 9

Delightra and her sister Gladell bought a new book together, and are arguing over who gets to read it first. Gladell offers to toss a coin, but Delightra is suspicious because Gladell seems to have been winning coin tosses an awful lot lately. Delightra says "Gladell, you're a cheat. Whatever I call, you know whether to catch it just a bit earlier or not to make sure that you win. Your coin always spins at the same rate, and you must know exactly what position to catch it." Gladell is outraged, and replies "I can't believe you think I'm cheating! Yes, my coin always spins at the same rate, but that doesn't make me a cheat. You're just jealous because I'm luckier than you!"

After complaining to her physicist mother, and making her mother observe one of Gladell's coin tosses, Delightra is told that it is completely implausible for Gladell to be cheating in this manner, as the coin rotates once for each coin-length it moves upwards. Which of the following is the best explanation of why?

(A) Although Gladell can flip the coin to the same height each time and accurately judge this height, quantum mechanics tells us that there will be some inherent uncertainty in the coin's and her hand's position, and the sum of these is greater than a coin length so the coin may flip an extra time.

(B) Even if Gladell tries to flick the coin exactly the same way each time, the small differences in the initial vertical velocity mean that the height of the toss isn't accurate to within a coin length, so it might flip an extra time before she catches it.

(C) Because the coin rotates once per coin length, the chaotic eddy currents in the air slow the coin's spin, so although Gladell can get the position right to within a coin length, the coin may have flipped an extra time.

(D) Although Gladell can flick the coin to the same height within a coin length each time, she cannot measure that constant height to within a coin length and so she can't calculate where to put her hand.

(E) Even if Gladell flicks the coin to the same height within a coin length each time, she will give it some slight horizontal velocity, so the time for which the coin is in the air changes and it might flip an extra time before she catches it.

Solution: (b) – the coin will reach a height of around 0.5 m in a good coin toss, so it will travel a total distance of 1 m. For Gladell to be able to cheat she must be able to keep the distance travelled constant to within well under a coin length, i.e. less than 1 cm. She would not be able to toss a coin that accurately.

Question 10

An equation is dimensionally correct if the quantities on both sides have the same physical dimensions, e.g. length (L), mass (M), time (T), and combinations. For example, distance, d, is a length and speed, v, is a length per unit time, so $d = vt$ is dimensionally correct if t represents a time. Equations cannot be physically correct unless they are dimensionally correct.

The average speed v of a particle in an ideal gas depends only on the mass of the particle m, the pressure P, and the number density (number of particles per unit volume) n of particles in the gas. Pressure has the dimensions $\mathrm{M\,L^{-1}\,T^{-2}}$. Which of the following equations is dimensionally correct?

(A) $v = \left(\frac{nm}{P}\right)^{1/2}$

(B) $v = \dfrac{P}{nm^{2/3}}$

(C) $v = \left(\dfrac{P}{nm}\right)^{1/2}$

(D) $v = Pnm$

(E) $v = \left(\dfrac{Pm}{n}\right)^{1/2}$

Solution: (c) – the dimensions of $v = \left(\dfrac{P}{nm}\right)^{1/2}$ are

$$\left[\frac{\mathrm{ML^{-1}T^{-2}}}{\mathrm{(L^{-3})(M)}}\right]^{1/2} = LT^{-1} \quad ,$$

which are the dimensions of speed.

Section B

Written Answer Questions
Attempt **ALL** questions in this part.

You may be able to do later parts of a question even if you cannot do earlier parts.
Remember that if you don't try a question, you can't get any marks for it — so have a go at everything!
Suggested times to spend on each question are given. Don't be discouraged if you take longer than this — if you complete it in the time suggested consider that you've done very well.

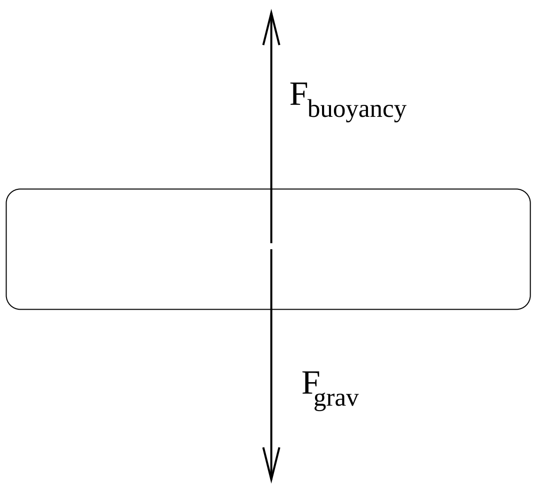
Question 11 *Suggested time: 15 minutes*

A solid object floats in water if its average density is less than that of water. If its average density is greater than the density of water it sinks. If its average density is equal to that of water then it has no nett force acting on it when it is submerged.

A boy floating in a swimming pool notices that he can float easily if he holds his breath but that as he breathes out, no matter how slowly he does it, he ends up just under water once he has finished exhaling. If he breathes out by blowing a fast stream of air directly upwards he goes under water even sooner and ends up further under water and is still moving down towards the bottom of the pool after he has finished exhaling.

The difference between the volume of air in his lungs when full and empty is V_l and he has mass m. The density of water is ρ_w and the density of air is much less than water.

(a) Draw a free body diagram showing the forces acting on the boy when his lungs are empty after breathing out slowly. Indicate the magnitude of the forces acting on the boy by the size of the arrows you draw to represent the forces.
Solution: There should be two arrows drawn from a central object: one upwards and one downwards. They should be equal in magnitude, indicating that there is no nett force on the boy. One arrow should be labelled 'F_{buoyancy}' and the other 'F_{grav}', or similar. A sample free body diagram is given below: (2 marks)



(b) What is the boy's average density when his lungs are empty? Explain your answer.
Solution: The boy's average density is ρ_w, the density of water. If it were greater, then he would sink, but this does not occur when he exhales slowly. If it were less, he would float, and thus not completely submerge. Therefore his average density

must be equal to that of water. (2 marks)

(c) What is the boy's average density when his lungs are full?
Solution: When the boy's lungs are empty, he has density ρ_w. When his lungs are full, his mass remains the same (as the density of air is negligible), but his overall volume has been increased by V_l.
Letting ρ_f be the boy's average density when his lungs are full, and V_e be the boy's total volume when his lungs are empty, we can thus calculate that

$$\begin{aligned} \rho_f &= \tfrac{m}{V_e+V_l} && (1)\\ \frac{1}{\rho_f} &= \tfrac{V_e+V_l}{m} && (2)\\ &= \tfrac{V_e}{m} + \tfrac{V_l}{m} && (3)\\ &= \tfrac{1}{\rho_w} + \tfrac{V_l}{m}, && (4)\\ & && (5) \end{aligned}$$

and so

$$\rho_f = \frac{m\rho_w}{m + V_l\rho_w} \quad .$$

(2 marks)

(d) Draw a free body diagram of the boy as he is moving downwards just after he finished emptying his lungs by blowing a fast stream of air upwards. Again indicate the magnitude of the forces acting on the boy by the size of the arrows you draw to represent the forces.
Solution: The diagram should be as in (a). The addition of a *small* arrow in the direction of F_{buoyancy} labelled as a drag force may be accepted.(2 marks)

(e) Why does the boy end up further under water in this case?
Solution: When the boy blows fast upwards, he is imparting a greater force to the air. Consequently, he receives a greater equal and opposite force himself. This impels the boy downwards.
When the boy exhales slowly, he expels the air with much less force. Because of this, he is pushed into the water only very slowly.
Thus, by expelling the air quickly he essentially propels himself into the water, which will then slow him only due to possible drag forces (there will be no net buoyancy force, since his density is equal to that of water). Because of the greater propulsion when breathing out quickly, he will end up further underwater.
(2 marks)

Marker's comments:

- Many submissions had the upwards force labelled as 'water resistance', or similar. The upwards force the boy experiences was due to buoyancy, which is quite different to any sort of drag force.
- A free body diagram should include only forces which are acting on the body in question; forces the boy exerts on the water should not be included.
- While the boy does end up underwater regardless of his exhalation speed, he is not sinking. No matter his exhalation speed, once he is fully submerged there are no nett forces upon him. In the case where he exhales quickly, this implies that he will then be drifting downwards at constant velocity.

- Many responses to part (c) simply stated that the boy's density would be less than that of water. While correct, a calculation was required for this section.
- In part (d) the question specifies that the free body diagram should be drawn *after* the boy has finished breathing out: thus the boy is no longer experiencing any additional downwards force due to his exhalation.
- The boy's rapid change in density will make some small contribution to his downwards velocity. However, the reaction force from his upwards exhalation will have the more significant effect. In part (e) students were expected to recognise this fact.
- An argument based on conservation of momentum was also accepted for part (e).

Question 12 *Suggested time: 25 minutes*

Ben has built a toy car. He has worked hard at reducing the friction in the wheels, so much so that it can be considered negligible. The car is, however, subject to a drag force $F_d = \kappa v^2$ where v is the speed of the car and $\kappa = 0.030\,\text{kg}\,\text{m}^{-1}$. The car has mass $m = 2.5\,\text{kg}$ and cross-sectional area $A = 0.06\,\text{m}^2$. Take the acceleration due to gravity to be $g = 9.8\,\text{m}\,\text{s}^{-2}$.

Ben places the car at the top of a ramp of length $s = 1.2\,\text{m}$ inclined at an angle $\theta = 30°$ to the horizontal. He pushes it for time $t = 0.25\,\text{s}$ with a force such that the nett force acting on the car is $F_p = 20.0\,\text{N}$. He then lets the car travel down the ramp and along a flat track.

(a) Consider the car at time t, when Ben stops pushing and find

(i) v_t, the speed of the car and
(ii) s_t, the distance the car has traveled.

Solution: Applying Newton's second law

$$\begin{aligned} F_p &= ma_1 \\ \frac{F_p}{m} &= a_1 \quad . \end{aligned}$$

Since the acceleration is constant

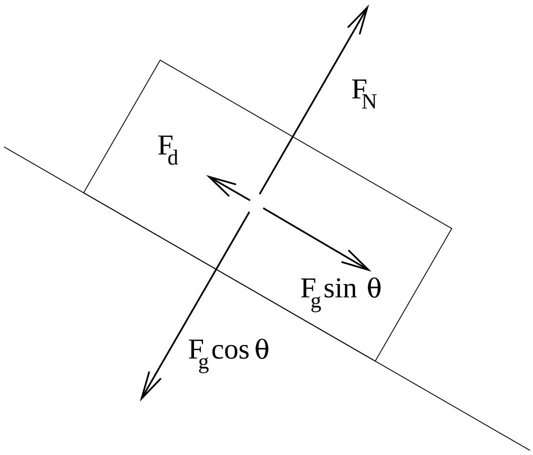
$$\begin{aligned} v_t &= a_1 t \\ &= \frac{F_p}{m} t \\ &= 2.0\text{ms}^{-1} \quad . \end{aligned}$$

We can relate the distance travelled to the initial velocity, $u = 0$, the acceleration and the time taken

$$\begin{aligned} s_t &= ut + \frac{1}{2} a_1 t^2 \\ &= 0 + \frac{F_p}{2m} t^2 \\ &= 0.25\text{m} \quad . \end{aligned}$$

(3 marks)

(b) Find an expression for v_b, the speed of the car when it reaches the bottom of the ramp. Drawing a free body diagram of the car will help you. Note that v_b is approximately $4\,\text{m}\,\text{s}^{-1}$.
Solution:



$$\begin{aligned} F &= F_g \sin\theta - F_d \\ ma_2 &= mg\sin\theta - \kappa v^2 \\ a_2 &= g\sin\theta - \frac{\kappa}{m}v^2 \end{aligned}$$

The second term on the right hand side of the equation will be less than approximately $\frac{0.030\text{kgm}^{-1}}{2.5\text{kg}}(4\text{ms}^{-1})^2 = 0.2\text{ms}^{-2}$. The first term is constant and is equal to $9.8\text{ms}^{-2}\sin 30° = 4.9\text{ms}^{-2}$, which is always much bigger than the second term, so we can neglect the second term. Hence, the acceleration of the car is approximately constant, so

$$\begin{aligned} v_b^2 &= v_t^2 + 2a_2s_2 \\ &= \left(\frac{F_p}{m}t\right)^2 + 2g\sin\theta\,(s - s_t) \quad , \\ v_b &= \sqrt{\left(\frac{F_p}{m}t\right)^2 + 2gs\sin\theta - \frac{gF_p}{m}t^2\sin\theta} \quad . \end{aligned}$$

(4 marks)

(c) After he has released the car, Ben notices that there is a large block on the track, a distance $d = 12.5\,\text{m}$ from the bottom of the ramp. Find v, the velocity of the car when it hits the block.

Solution:

$$\begin{aligned} -F_d &= ma_3 \\ \frac{-\kappa}{m}v^2 &= a_3 \end{aligned}$$

This equation has the same form as the one given in the useful information, with $k = \frac{-\kappa}{m}$, so

$$\begin{aligned} v_b &= le^{\frac{-\kappa}{m}0} \\ &= le^0 \\ &= l \quad . \end{aligned}$$

So we have

$$v = v_b e^{\frac{-\kappa}{m}x}.$$

We can use our previous expression for v_b and substitute $x = d$ to find the car's velocity when it reaches the block

$$\begin{aligned} v &= \left[\sqrt{\left(\frac{F_p}{m}t\right)^2 + 2gs\sin\theta - \frac{gF_p}{m}t^2\sin\theta}\right] e^{\frac{-\kappa}{m}d} \\ &= 3.1\text{ms}^{-1} \end{aligned}$$

(4 marks)

Useful Information

If the acceleration of a body, a, is related to the velocity by $a = kv^2$, the velocity is related to the position, x by $v = le^{kx}$ for some constant, l.

In physics it is often useful to make approximations. This can simplify your calculations, and if the approximation you make is appropriate, it won't change your result appreciably. For example, if you know that $A = B + C$ and that C is much, much smaller than B, you may be able to say that $A = B$ and get the same result as you would have using $A = B + C$. If you make an approximation you must demonstrate that it is valid.

Marker's comments: Part (a) was well done by many students, however a large number of students treated F_p as the force applied by Ben rather than the nett force. A number of students tried to use conservation of energy for this and other parts. Most did this by equating the initial gravitational potential energy with the final kinetic energy without considering the nature of the forces at work. In part (ii), many students assumed that the velocity of the car was constant. This was not correct.

Part (b) could be done using either the method above or conservation of energy. Many students attempted one of these methods but most failed to explain why the drag force could be neglected. Some students who did explain that the drag force was much smaller than the gravitational force assumed that the drag was constant and used the value of v_t they found in part a or the approximate value of v_b given in the question to calculate F_d. This was accepted if the student gave adequate explanation of why the change in the force was negligible. A number of students didn't relate this part to the previous part and assumed that the car began stationary and/or at the top of the ramp rather than using the expressions they had already found. The overwhelming problem with responses to this part was that almost no students gave an algebraic expression for v_b. Most students who completed this part gave a numerical answer instead.

Part (c) was not very well done by many students. Many students continued to neglect the drag force or treated it as a constant. Of the students who used the correct expression for the drag force many confused κ in the useful information with k, giving them a dimensionally incorrect expression and an incorrect answer.

A surprising number of students mislabeled the parts of this question. Many students labeled part (b) as (a) (iii). More surprisingly, a number of students labeled part (c) as part (d).

Question 13 *Suggested time: 15 minutes*

Elizabeth wants to find the gravitational field strength in her bedroom using a pendulum. The equation she will use assumes that the pendulum consists of a massless string attached at one end to a fixed point and at the other to a point mass, that is a mass that has no volume. It also assumes that the angle between the string and the vertical is always small.

She takes a hair ribbon, about 20 cm long, measures it with a ruler and then ties one end to her hair brush. She realises that she doesn't have enough hands to do the experiment on her own, so she asks her sister Claire to hold the other end of the ribbon. Elizabeth asks Claire to rest her arm on a book case to make sure that she holds the ribbon very still. This forms Elizabeth's pendulum.

Elizabeth pulls the hair brush to the side and counts that it passes four thick books as she pulls it. She then releases it and lets it swing. She gets out her mobile phone and uses the stopwatch on it to time the pendulum. She times ten periods instead of one because she thinks this will be more accurate. She watches to see when the pendulum reaches the fourth book because it is moving very slowly here so she has more time to press the button on her phone.

Thinking that she'll get a better result if she takes more data and plots a graph, she repeats the experiment with ribbons of different lengths, pulling it past the same four thick books before letting it go each time.

(a) Consider Elizabeth's experimental method. What did she do that could have adversely affected her results? Suggest a simple modification to the experiment that could solve each problem.

Solution: There are a number of factors likely to affect the results. Some are:

- The angle made by the ribbon with the vertical is too high, particularly for a shortish ribbon, and should be reduced for the theory to accurately describe the experiment.
- The timing was performed at the end of the swing, which is the least accurate place as there is a large uncertainty in the time at which it reaches the end. The periods should be timed from the middle of the swing.
- The ribbon was measured before tying it to the brush. Even for a modestly round hair brush, tying the knot is likely to use up some centimetres. The ribbon should be tied on before its length is measured.
- The hairbrush is not a good approximation to a point mass, because it is likely to swing in other dimensions, so the pendulum will have more degrees of freedom than it is supposed to, and the simple theory will not describe its motion accurately. It would be far better to replace it with a small, compact object, such as a marble or a pendant.
- The use of someone's hand as the pivot point is likely to introduce error. Not only will the person's fingers potentially slip on the ribbon, but her hand is unlikely to stay still throughout the experiment, and motion of the pivot can seriously affect the swing of the pendulum. The ribbon should be fixed with a thumbtack or tied to a hook.

These were the main suggestions that received substantial credit. Of course there were many other suggestions, and they were marked on their merits. (6 marks)

(b) Estimate the percentage uncertainty in the length for a 20 cm ribbon and for a 1 m ribbon using Elizabeth's original method. Repeat the estimate for your revised method, if it is different.
Solution: The critical factor in the uncertainty here is the length of the ribbon. For a 20cm ribbon, some centimetres are likely to be lost in tying the knot, and in the holding of the string, so an error of 8cm, i.e. about 40%, seems like a reasonable estimate if the initial measurement is taken as the value. For a 100cm ribbon, the same absolute uncertainty is only around 10%. Note that reasonable estimates were given credit, students did not have to obtain the numbers presented here. However, assuming that the only uncertainty was due to the scale on the ruler was not realistic and not given credit. Students who estimated the error for their revised method, giving reasons for the changes, and obtained a reasonable result were given credit for the second part. (4 marks)

Marker's comments: Most students identified at least one of the major sources of error in the experiment, with the best students getting most of them. The most common factor listed that was not considered significant was air resistance due to the width of the ribbon, which is insignificant compared with the other factors here. Some students claimed that the decay of the pendulum's amplitude meant that timing multiple periods was a bad idea; this is not the case, as to a first approximation the period does not depend on the amplitude. After ten periods it is certainly plausible that the pendulum is still oscillating, which was questioned by some students. Overall the responses to this part indicated that students were thinking about multiple sources of error, and the suggestions for how to improve the experiment were generally good.

The second part was not attempted by a significant number of students. Many students who did write something simply wrote a number with either no reasoning at all, or a statement that it was 'about this big' with no justification. This did not earn many marks. The most common mistake made by those who did provide some reasoning was in assuming that the entire uncertainty was due to the scale of the ruler, which is a poor assumption. Students who calculated sensible values for the method as presented in the paper were not always able to obtain sensible values for a revised method.

Question 14 *Suggested time: 30 minutes*

Joseph wants to measure the *latent heat of vaporization* of substance X, which is the amount of energy required per unit mass to turn substance X from a liquid to a gas at its boiling point. He begins by putting approximately 200 mL of liquid X in a beaker on a combined hotplate and scale. He inserts a thermometer, turns on the hotplate at $t = 0$ min and records the liquid's temperature as well as the combined mass of the liquid, beaker and thermometer every minute. After 24 minutes, there is no more liquid in the beaker. Joseph's results are shown in the table below.

Time (min)	0	1	2	3	4	5	6	7	8
Temperature (°C)	24	25	28	37	50	64	77	90	102
Mass (g)	310	310	310	310	310	310	310	310	310

Time (min)	9	10	11	12	13	14	15	16	17
Temperature (°C)	113	124	134	143	152	158	160	160	159
Mass (g)	310	310	307	307	305	302	288	264	241

Time (min)	18	19	20	21	22	23	24
Temperature (°C)	160	160	160	161	160	161	-
Mass (g)	214	190	165	138	110	79	69

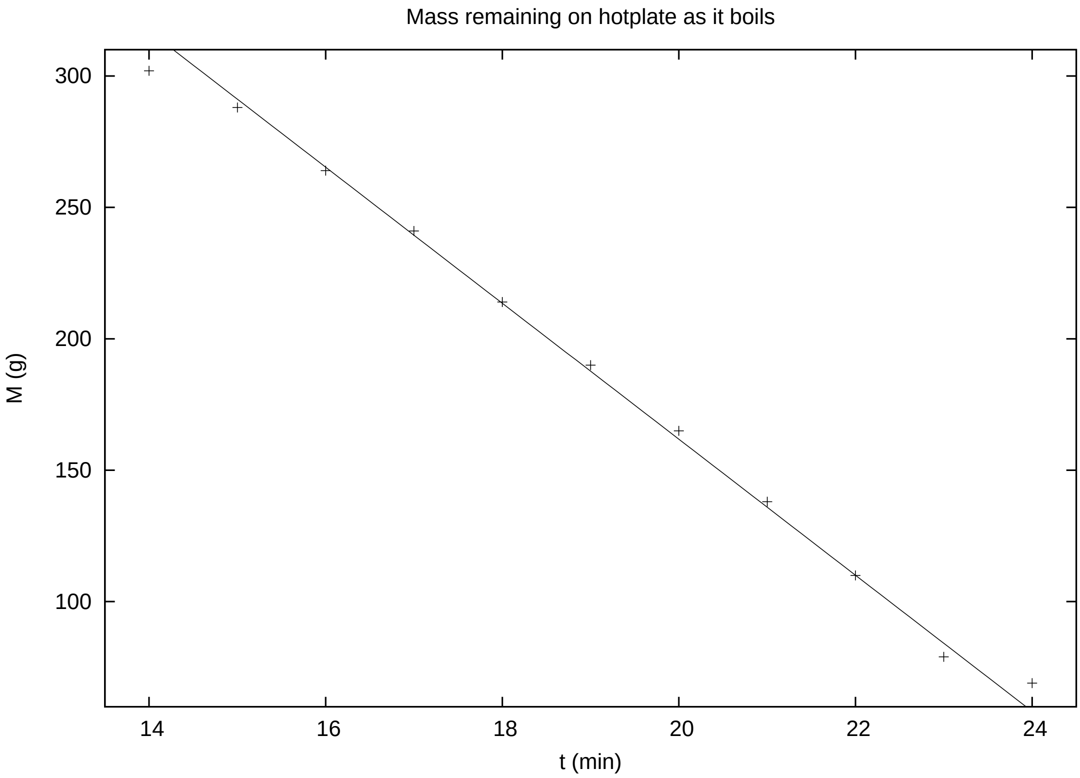
Joseph notices that the temperature increases at a different rate at different temperatures. To show this variation, he plots the difference between two successive temperature measurements divided by the time between them, i.e. the rate of change of the temperature, versus the temperature at the first of these measurement times. His graph (Graph 1) is shown over the page, along with a graph (Graph 2) of the temperature of the liquid as a function of time.

(a) Explain the shape of Graph 1, by giving reasons for the initial increase, the gradual decline over a wide temperature range and the sharp decrease near 160 °C. (Hint: Graph 2 might help you interpret the regions of Graph 1.)

Solution: The initial increase occurs as the hotplate takes some time to warm up and the rate of heat flow to liquid X increases as it gets warmer. The gradual decline occurs as the temperature of liquid X increases because more heat is lost to the surroundings when it is at higher temperatures. The sharp decrease occurs as once the liquid reaches its boiling point all the heat energy goes to converting the liquid to a gas, rather than increasing its temperature. (3 marks)

(b) By drawing an appropriate graph, find the average rate at which liquid X boils, in $\mathrm{kg\,min^{-1}}$.

Solution: The slope of the line of best fit to the nearly linear region of the following graph of mass remaining versus time is the negative of the average rate of boiling. Note that plotting data for times when the liquid is not boiling gives no additional information and is a waste of time.



This gives the average rate of boiling to be $2.6 \times 10^{-2}\,\mathrm{kg\,min^{-1}}$. (7 marks)

(c) In a previous experiment, Joseph measured the specific heat capacity of liquid X to be 2.19 $\mathrm{kJ\,kg^{-1}\,K^{-1}}$. This means that it takes 2.19 kJ of energy to raise the temperature of 1 kg of liquid X by 1 K (which is equal to 1 °C).

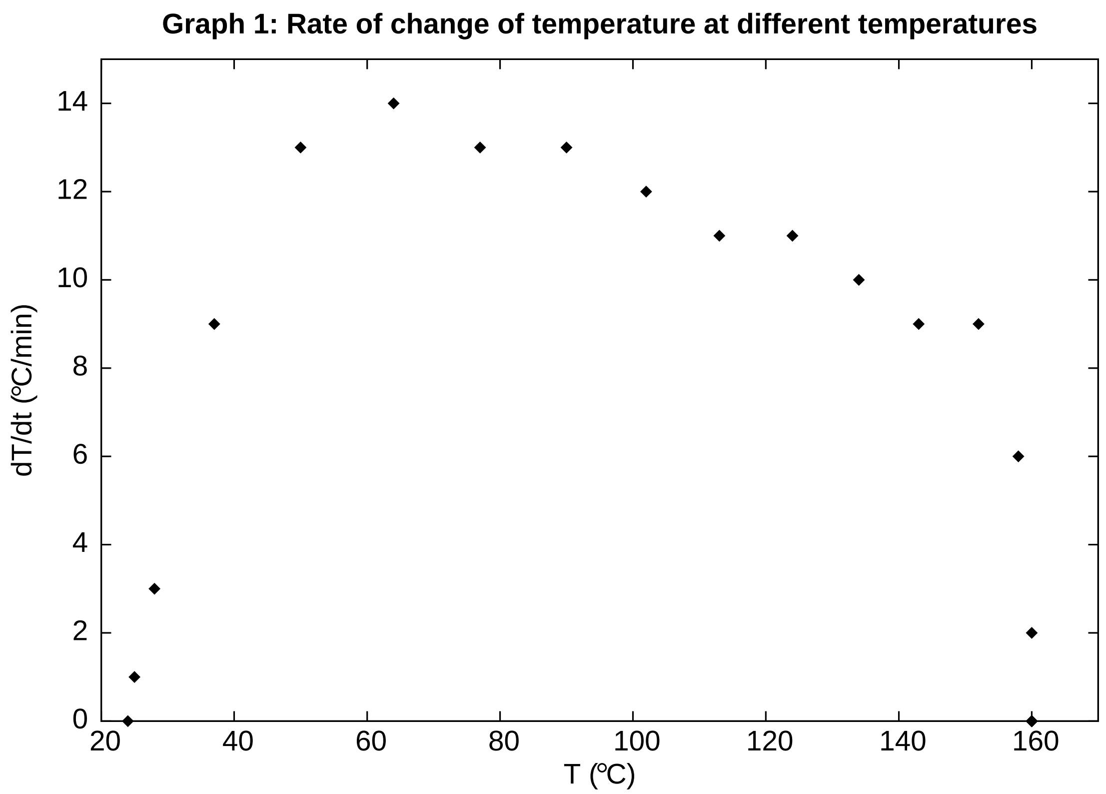
(i) Pick the point on Graph 1 that gives the most information about the rate at which heat is transferred to the liquid at its boiling point. Explain how you chose this point and why it is the most appropriate.
Solution: The point at $T = 152\,°\mathrm{C}$ and $dT/dt = 9\,°\mathrm{C\,min^{-1}}$ as the liquid has not begun boiling but the rate of heat loss to the surroundings is closest to that at the boiling point. (3 marks)

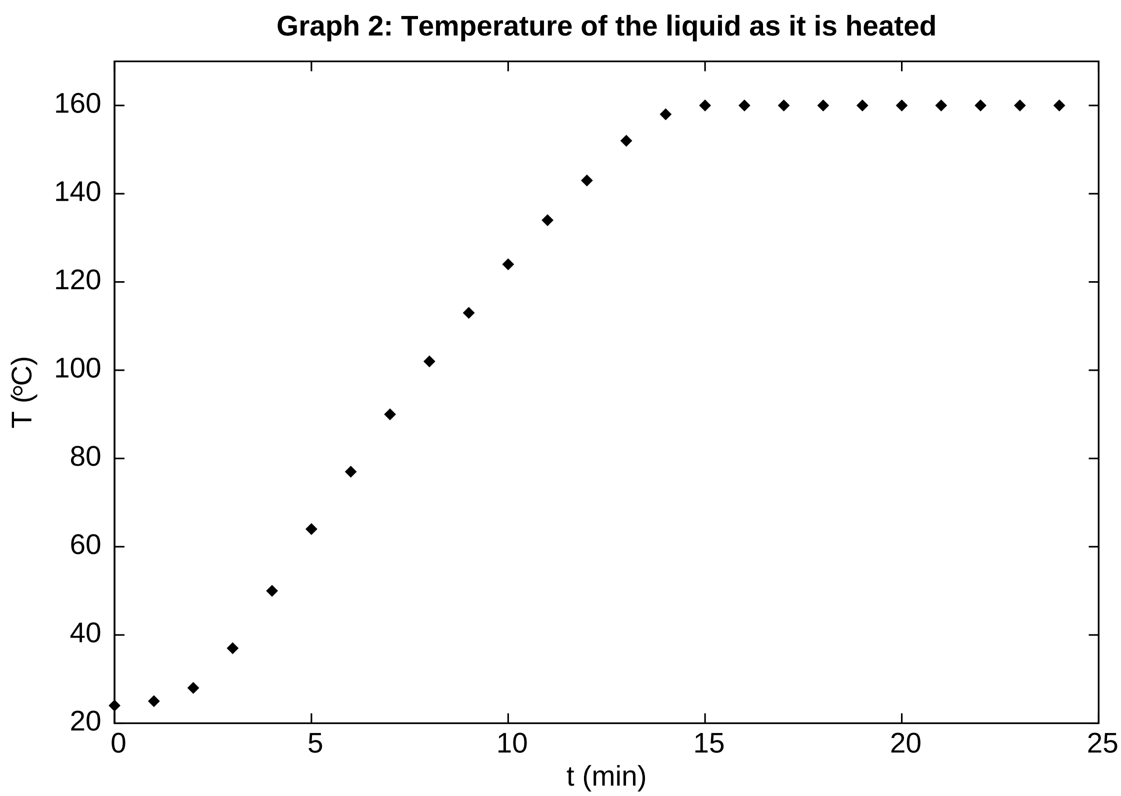
(ii) Using the information above and your answer to part (i), find the power being transferred to liquid X as it boils, in $\mathrm{kJ\,min^{-1}}$.
Solution: The combined mass of the beaker and thermometer is 69 g so the mass of liquid at this temperature is m_X =236 g. The power transferred

$$P = m_x c \frac{dT}{dt} = 4.7\ \mathrm{kJ\,min^{-1}} \quad ,$$

where c is the specific heat capacity of liquid X. (2 marks)

(d) Using your answers to the previous parts, find the latent heat of vaporization of substance X, in $\mathrm{kJ\,kg^{-1}}$.
Solution: The latent heat of vaporization is the power divided by the rate of boiling, so $L = 180\,\mathrm{kJ\,kg^{-1}}$. (1 mark)


Graph 1: Rate of change of temperature at different temperatures
dT/dt (°C/min)
14
12
10
8
6
4
2
0
20
40
60
80
100
120
140
160
T (°C)


Graph 2: Temperature of the liquid as it is heated
T (°C)
160
140
120
100
80
60
40
20
0
5
10
15
20
25
t (min)

Marker's comments: Most students attempted at least one of parts (a) or (b) of this question, however many did not attempt more than one part. Many students could not interpret the supplied graphs correctly for part (a); the most common error in interpretation was equating a changing rate of change of temperature with a change of temperature. Of those who could correctly interpret the graphs, many students gave a correct explanation for one of the regions, but then incorrectly applied the same explanation to the other regions.

In part (b) most students plotted more data than was required and if they drew a line of best fit tried to fit to too many points. Other students didn't use their graphs to calculate the boiling rate. It was also noted that many students could not plot points accurately.

For parts (c) and (d) the greatest difficulties were with understanding the ideas of specific heat capacity and latent heat and how to apply conservation of energy.

Question 15 *Suggested time: 20 minutes*

The intensity, power per unit area, of light falling on a surface in three dimensions can be understood by thinking about light rays coming from a point source in all directions. The total power of the light falling on a given surface is proportional to the number of rays hitting that surface. The intensity at some point is therefore proportional to the density of the rays, or how many rays pass through a given area. The total number of rays leaving the point source depends only on the total power emitted by that source.

One of the consequences of this is that the intensity due to a single point source is inversely proportional to the square of the distance from the source,

$$I \propto P_0 \frac{1}{r^2} \quad , \tag{6}$$

where P_0 is the power emitted by the source. This is because at radius r, a constant number of rays must be spread over an area of $4\pi r^2$ (the surface area of the sphere of radius r), giving a density of rays proportional to $P_0/(4\pi r^2)$.

(a) How would intensity depend on distance in a universe with only two spatial dimensions, like a sheet of paper? Explain your answer.
Solution: The information given indicates that the intensity is proportional to the number of rays passing through a given surface. In a universe with only two spatial dimensions, there are only two dimensions in which the rays may spread, and so the appropriate equivalent of the spherical surface is the boundary of a circle. Since a circle has circumference $2\pi r$, and the intensity is proportional to the power divided by this surface, $I = P/2\pi r$. Thus the required proportionality is $I \propto 1/r$ in this universe. An appropriate diagram for the spreading of the rays is a useful addition.
(5 marks)

This concept works for some other physical quantities, such as electric field lines. The same arguments apply; the strength of the electric field due to a single point charge is inversely proportional to the square of the distance from the charge Q. The electrostatic force on an object with charge q due to the electric field of a point charge Q is

$$F = \frac{qQ}{4\pi\epsilon_0 r^2} \quad , \tag{7}$$

where ϵ_0 is a constant. Charges of opposite sign attract, and charges of the same sign repel one another.

(b) An object will move in uniform circular motion only if a centripetal force (i.e. a force acting towards the centre of the circle) is applied. The size of this force must be

$$F = \frac{mv^2}{R}, \tag{8}$$

where m is the mass of the object, v is its speed and R is the radius of the circle.
If an electron with mass m_e and charge $-e$ orbits a helium nucleus with charge $2e$ at radius R, what will the orbital speed of the orbit be? Assume that the orbit is circular.
Solution: In order to move in a circular orbit, the electrostatic force on the electron must provide the centripetal force for the orbital speed,

$$F = \frac{m_e v^2}{R} \quad .$$

Using the expression above for the magnitude of the force on the electron,

$$F = \frac{2e^2}{4\pi\varepsilon_0 R^2} \quad ,$$

and simple rearrangement of the above equations yields

$$v = \sqrt{\frac{e^2}{2\pi\varepsilon_0 m_e R}} \quad .$$

(2 marks)

(c) If two electrons are orbiting the helium nucleus, still at radius R, will their orbital speed be larger or smaller than that of a single electron orbiting? Explain your answer.
Solution: The two electrons are assumed to be orbiting at the same distance R. The only effect that they could have on the orbital speed is due to their interaction, as each experiences the same force due to the nucleus. Modelling the electrons as point charges, the force due to each electron on the other will be small and outward, assuming that the electrons spend time on opposite sides of the nucleus. Hence the nett force on each electron towards the nucleus is reduced, and so the velocity must also be reduced if the electrons are to remain in circular motion. An appropriate diagram showing this is a useful addition to the solution. (4 marks)

Marker's comments: a) A good fraction of students got the basic idea of the reduction in dimensionality correct. Of these students the most common errors were understanding that a circle was important, but failing to correctly identify the boundary. Some students claimed that it was the area of the circle, and not its circumference, which bounded the rays, leading to the incorrect proportionality. Some students wrote that the circumference was πr^2, again leading to an incorrect conclusion. Many students obtained the correct result, with varying degrees of explanation; the top students were able to clearly explain the logic, whereas some other students had difficulty writing a clear argument.

b) This part required the students to identify and substitute the correct charges into the expression given for the electrostatic force, and then to equate it to the given centripetal force. Some simple algebra then provided the solution. Most students were able to equate the two forces, but a surprising number did not arrive at the correct answer. Some students substituted only one charge, dropping the other factor of e completely. The most common mistake was carelessness rearranging the equation — many students lost terms from one line to the next and did not check that their solutions made physical sense.

c) This part required students to recognise that the repulsion between the electrons would slightly reduce the overall centripetal force and hence the velocity. Responses were mixed, with those students who used this idea making a sensible argument and obtaining most of the marks. Many students said that this effect did not occur or did not matter. Some students claimed that the velocity would increase, as there was 'more force': they did not take into account the fact that the new force was in the opposite direction. Some students also made the statement that the velocities would not change because the charge would double but the mass would also double, cancelling the effect. This is not sensible as the electrons are not a single particle and the force due to the nucleus on one particle does not change if an extra electron is added.